LABORATORY OF APPLIED MATHEMATICAL PROGRAMMING AND STATISTICS

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Distributionally Robust Transmission Expansion Planning: the Role of Uncertainty Quantification

FGV-Rio, July 10th 2019

Stochastic Programming Models and Algorithms for Energy Planning

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In this talk

- Motivation
- Multi-scale uncertainty
- Distributionally robust TEP model
- Enhanced column-and-constraint-generation algorithm
- Numerical experiments and discussion

Optimal plan property

Investment cost

Measure of the operational cost: $H(x)$

$$\underset{x, y(\xi)}{\text{minimize}}\ c_{inv}(x) + H(x)$$

subject to: $Ax \leq b$

$x \in \{0,1\}^n$

Investment constraints

Operational problem

$$H(x) = \min_{y(\xi)} \rho_{\xi}\{c(x, y(\xi), \xi)\}$$

Operational constraints

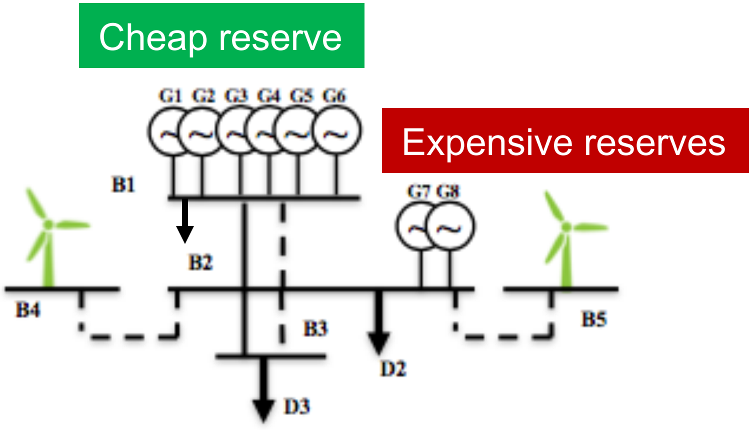
subject to: $Wy\big(\xi(\omega)\big) \geq h\big(\xi(\omega)\big) - Tx \quad \forall \omega \in \Omega$

- The optimal investment plan is such that the operational benefits and requirements are in equilibrium with the investment cost: $\partial c_{inv}(x) = -\partial H(x)$
- Question: how to teach the operational model all the economic benefits of transmission assets in the many possible circumstances?

Complex interactions example

It is key to consider complex interactions between substitute and complementary resources

- Renewable generation and reserve levels
  - Connecting renewables requires more reserves
- How to capture this complex interaction?
  - Uncertainty modeling plays a key role!

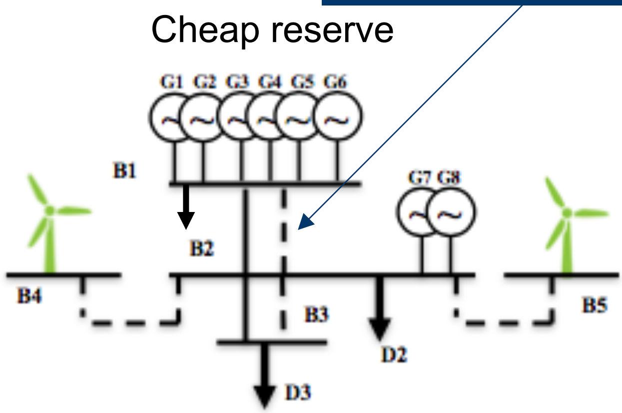


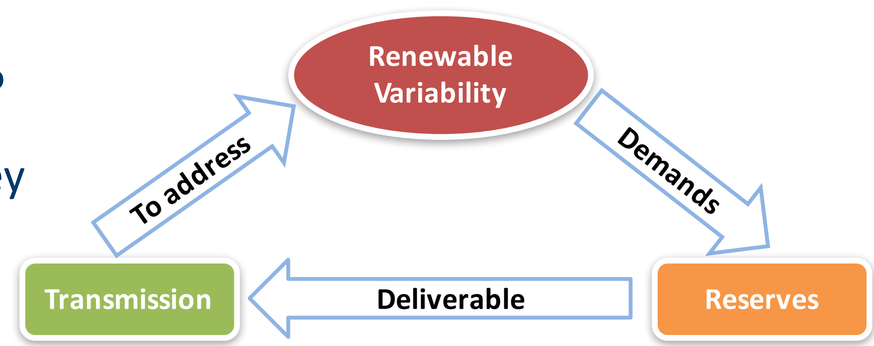
Complex interactions example

It is key to consider complex interactions between substitute and complementary resources

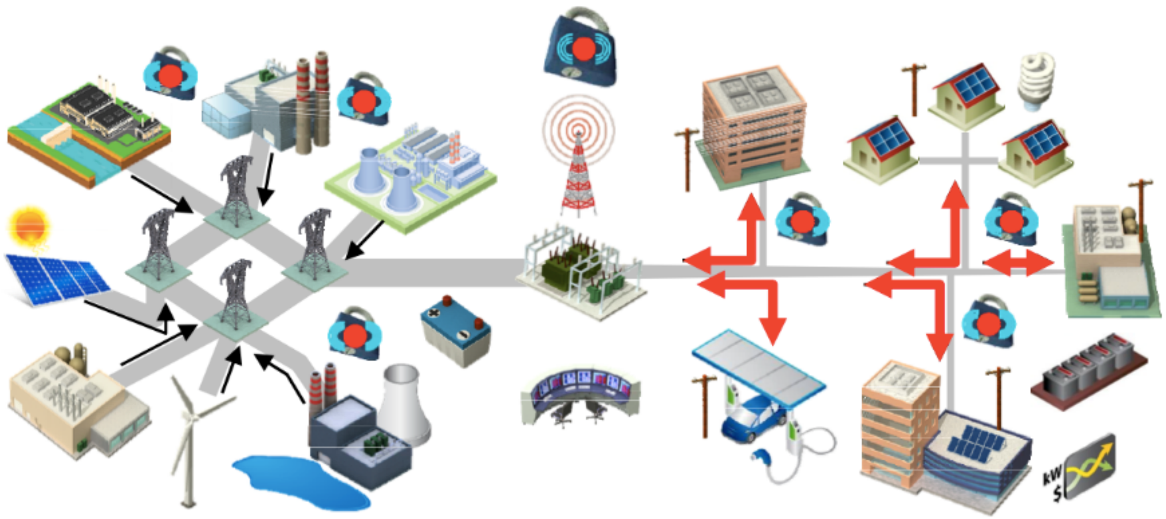
- New transmission lines may avoid expensive reserve deployment and ensure deliverability
  - New lines can bring cheap reserves from other areas
  - Voltage Kirchhoff's Law (KVL) and security criteria constraints must be considered

- How to capture this complex interaction?
  - Short-term operation modeling plays a key role!





Long-term uncertainty



21^{th} Century power system

- Market-driven active demand
- Large number of small non-controllable random power sources
- Energy storage devices smoothing or gambling ?
- Interactions with other networks and infrastructure (heat, gas, transport)

Deep uncertainty in long-term drivers

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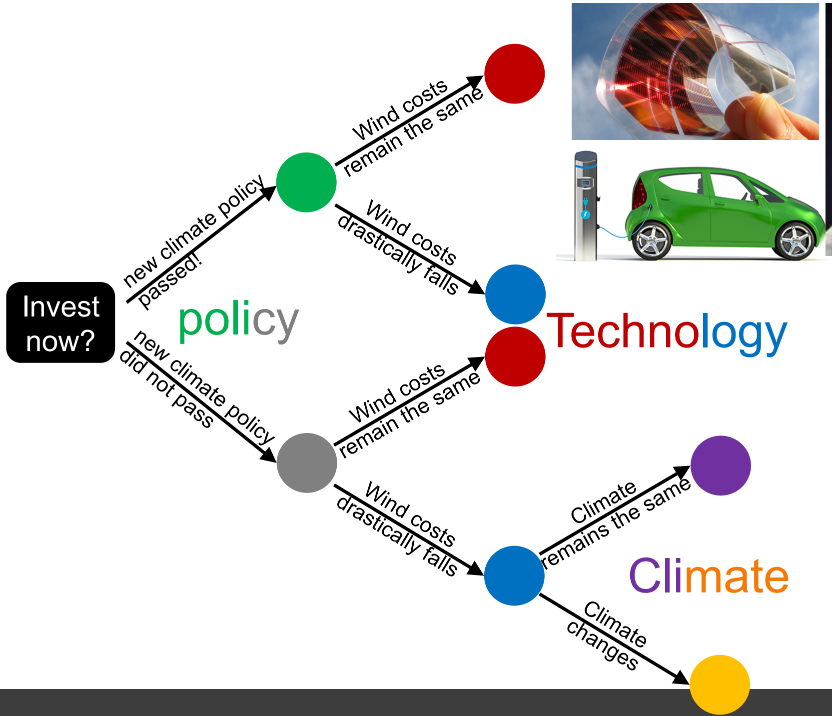


Global Warming's Potential Impact on Wind Energy

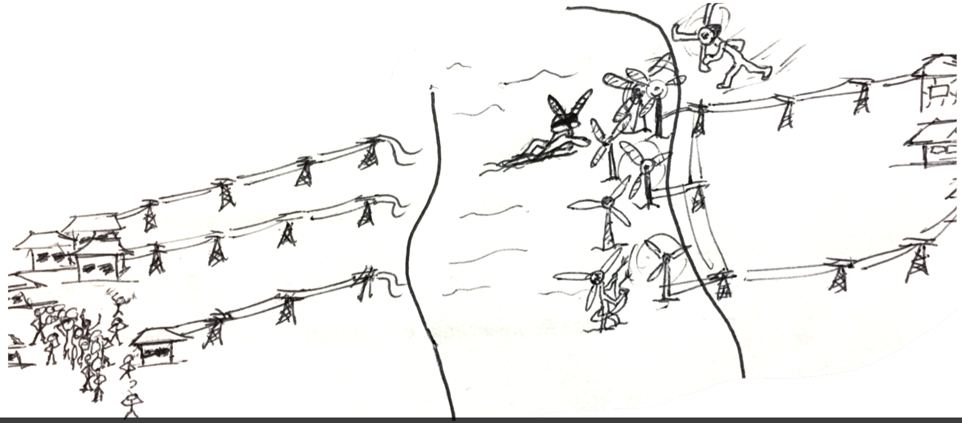
Published: November 17th, 2010

Three things you need to know:

-- Increasing global average surface temperatures may reduce the amount of wind energy available for electricity production, a new study says.

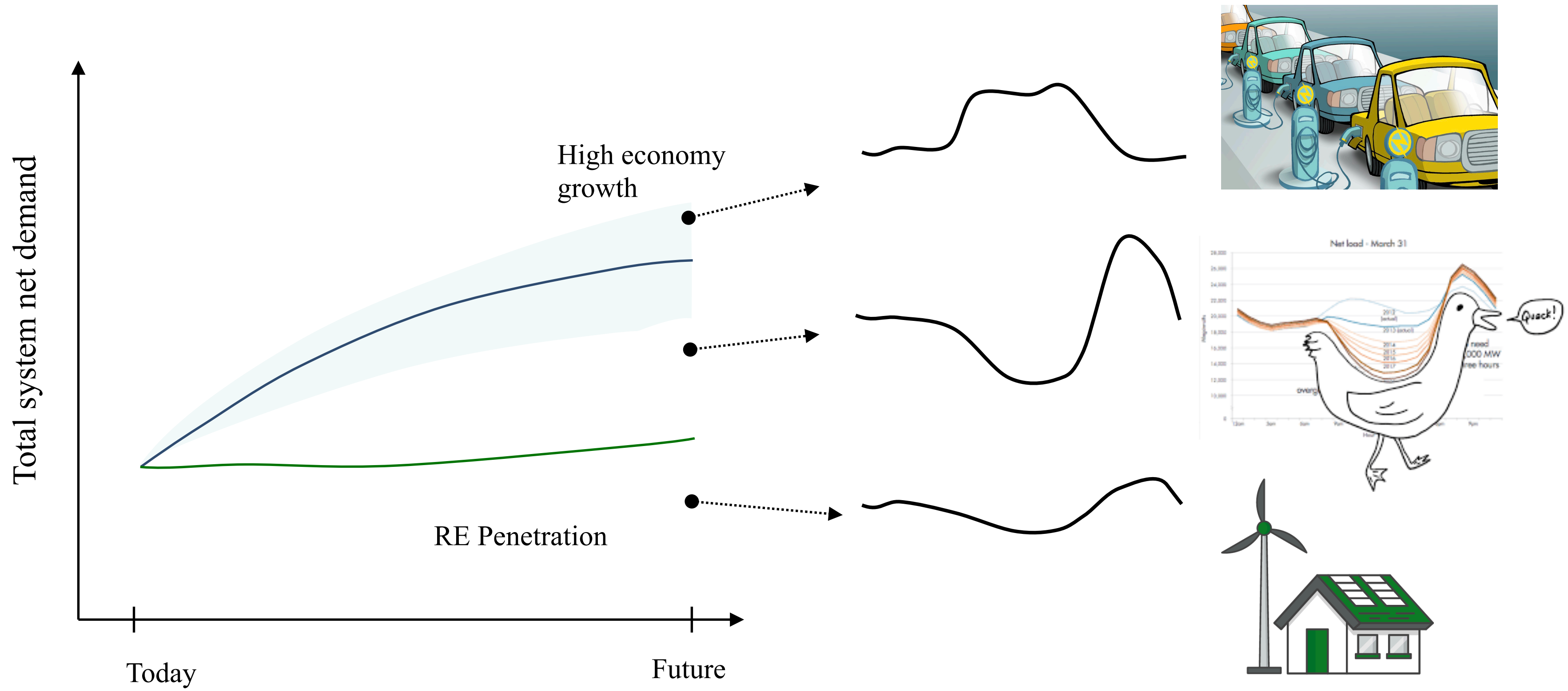
-- Global air circulation is influenced by many aspects of the global climate, including differences in atmospheric temperatures between the equator and the North and South Poles.

-- Across North America, average wind speeds have decreased slightly over the past 40 years.



- Deep uncertainty in economical, political, technological, climatic drivers
- In Brazil, the water inflows have significantly changed in the last years
- Policy makers can significantly change the incentives for renewables
- Technological disruptions are imminent !

Long-term drivers change short-term profiles



- Long-term scenarios define expected values (first moment) and ranges (support)
- Short-term uncertainty should be modeled as conditional distributions
- How to model the distribution of short-term not seen before?

Example of two-stage robust models

- Expanding lines and renewables to meet targets
- Co-optimization of generation, transmission, and reserve levels
- Compound security criterion:
  - n-1 and n-2 with zero load sheading
  - n-3 with no more than 2.5% load sheading
- With correlation between renewables generation

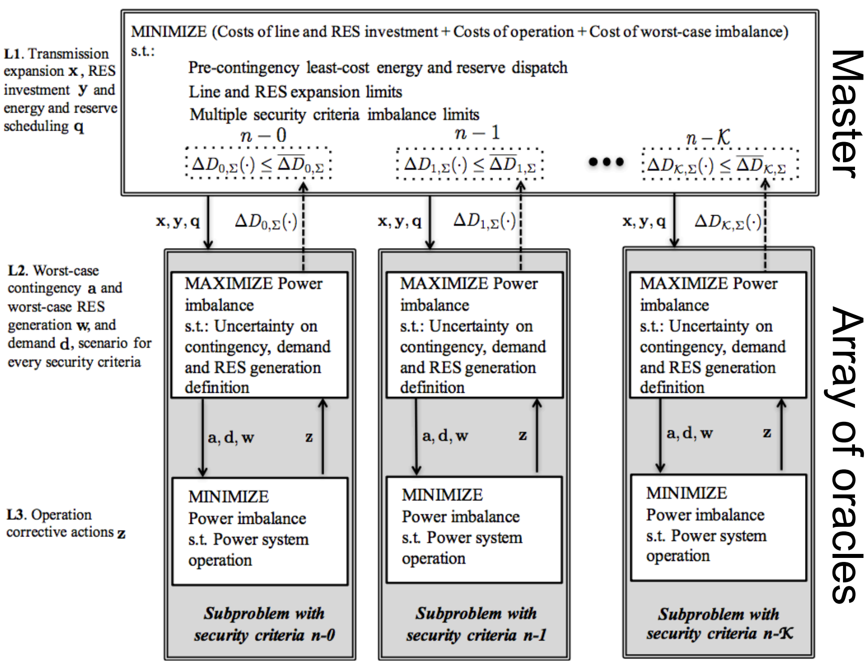


Fig. 1: Three-level robust TEP framework

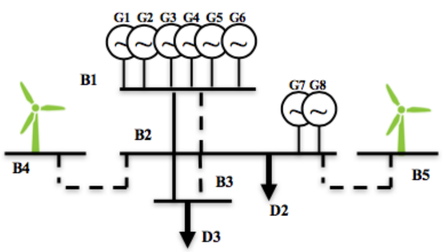


TABLE I: Total Reserve Cost ($/hour)

Correlation	Security Criteria			
	$K(0)$	$K(0 \to 1)$	$K(0 \to 2)$	$K(0 \to 3)$
-100%	0.00	162.00	349.00	1069.22
-50%	95.99	282.19	492.79	6992.92
0%	111.38	300.88	515.38	7945.11
50%	177.50	384.60	654.30	11336.40
100%	248.50	480.50	3693.50	14845.60

TABLE II: Total Investment in Lines ($/hour)

Correlation	Security Criteria			
	$K(0)$	$K(0 \to 1)$	$K(0 \to 2)$	$K(0 \to 3)$
-100%	6704.62	13760.30	21477.00	29307.80
-50%	7916.44	15064.90	22652.90	28361.10
0%	7954.40	15122.20	22791.30	28043.30
50%	7990.86	15159.90	22859.90	29151.10
100%	9319.00	16374.50	22947.00	29330.40

TABLE IV: Out-of-sample Monte Carlo Simulation Test for the Chilean Power System

Security Criteria	$K(0)$	$K(0 \to 1)$	$K(0 \to 2)$	$K(0 \to 3)$
LOL Interval	LOL Probability			
=0%	11.77%	85.72%	93.72%	96.88%
(0-1]%	7.65%	2.94%	2.22%	0.87%
(1-2]%	15.99%	4.35%	1.92%	1.10%
(2-3]%	15.44%	2.94%	1.10%	0.60%
(3-4]%	13.84%	1.86%	0.56%	0.24%
(4-5]%	10.25%	1.12%	0.23%	0.19%
(5-10]%	21.81%	1.04%	0.25%	0.12%
>10%	3.25%	0.03%	0.00%	0.00%
Expected LOL	3.49%	0.34%	0.12%	0.06%
CVaR of the LOL	11.13%	4.20%	2.16%	1.23%
Expected Total Costs [K$]	410.01	268.82	269.19	272.46
CVaR of the Total Costs [K$]	746.52	442.48	361.11	330.38

Main Chilean Power System Case Study

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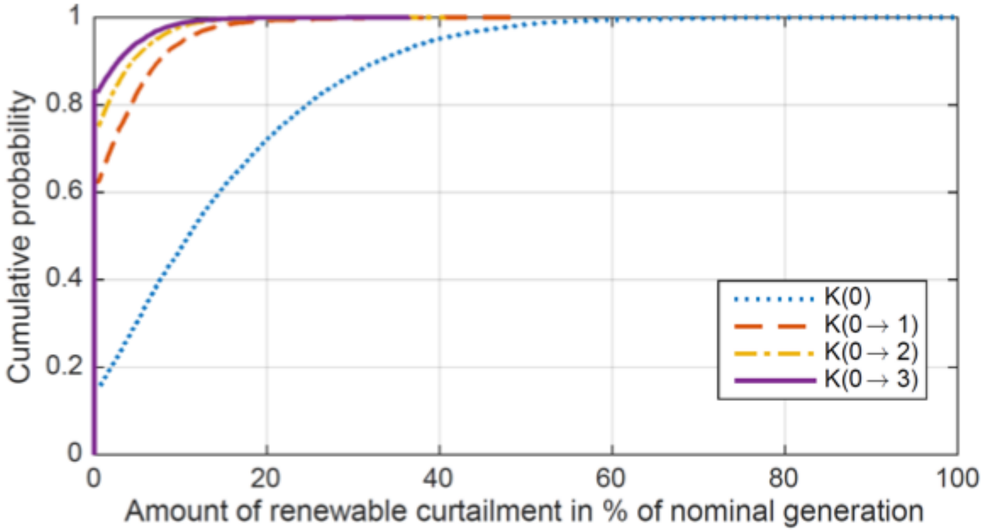
Main Chilean Power System Case Study

Wind spillage is mitigated while increasing security



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Long-term drivers change short-term profiles

Academic approach

- Hypothesis and principles
- Probabilistic models
- Data-driven estimation
- Many scenarios describing the estimated probability distributions (MC methods)

Industry approach

- Hypothesis and beliefs
- Big-data environment
- Collaborative panel of experts and long-term studies
- A few bottom-up economically coherent built scenarios

How do we teach models the many benefits of a transmission asset?

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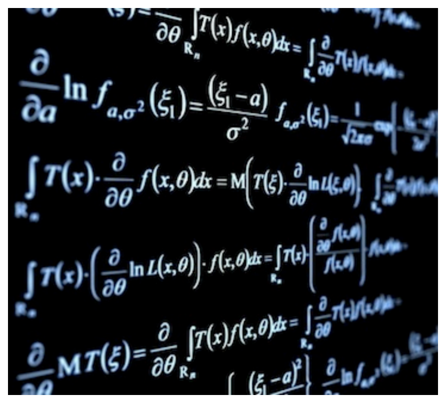
This is the goal of Operations Research

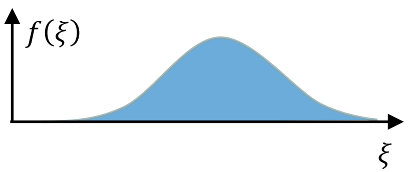
1. We need to define all possible actions and the level of information for each stage: planning and operational (variables)
2. We need to teach them how our world functions and what we accept or not through objective mathematical expressions (constraints)
3. And we need to define what is a virtuous plan (metric in objective function)



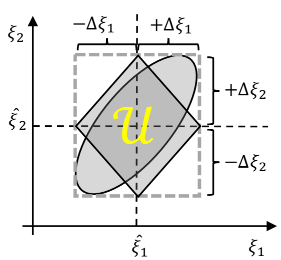
- We need to account for the uncertainties that create operational diversity in the short-term, allowing the model ”feel” the cost-and-benefit of each asset

Classic models for short-term uncertainty

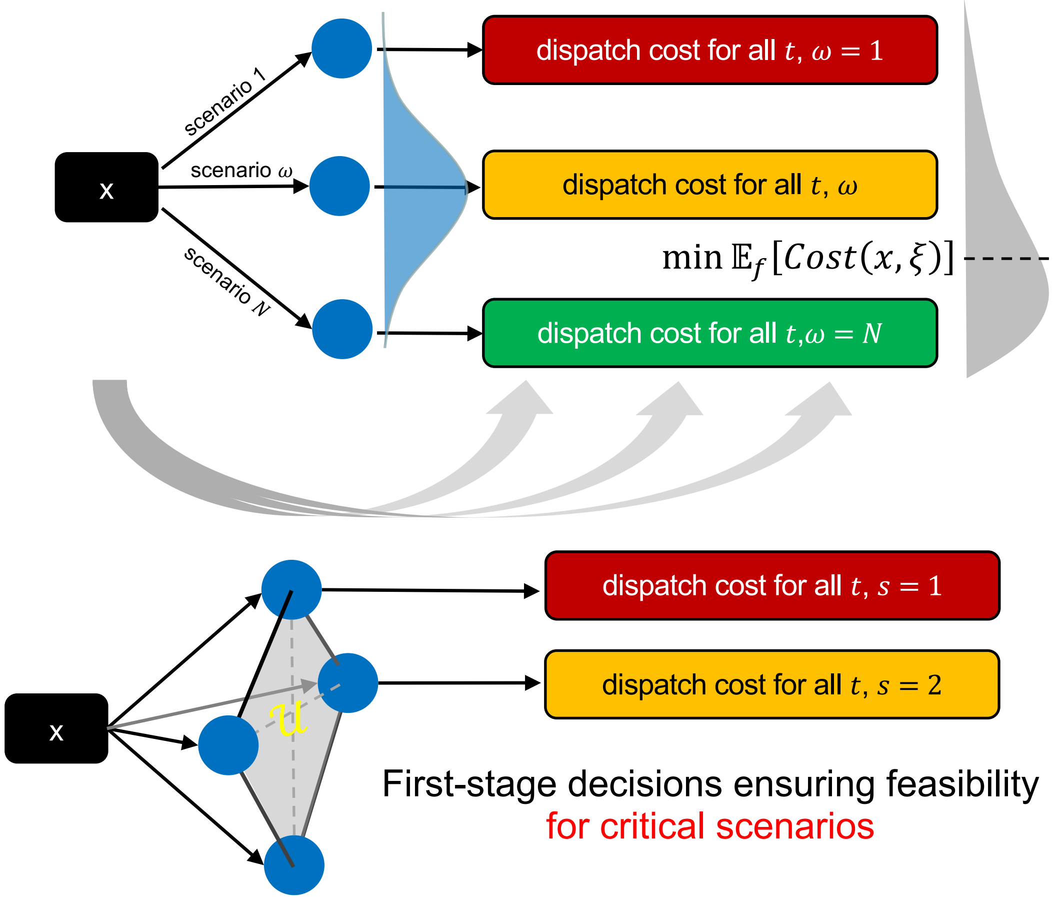
- Two-stage stochastic



- Two-stage robust

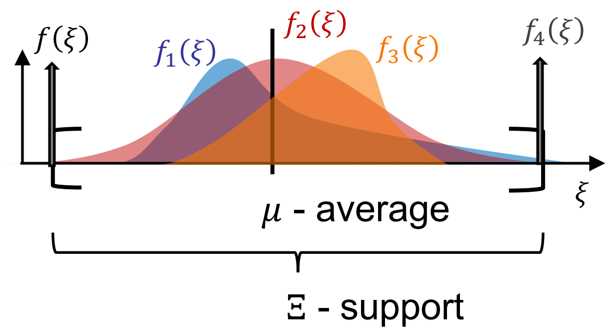


First-stage decisions minimize the average cost considering all scenarios

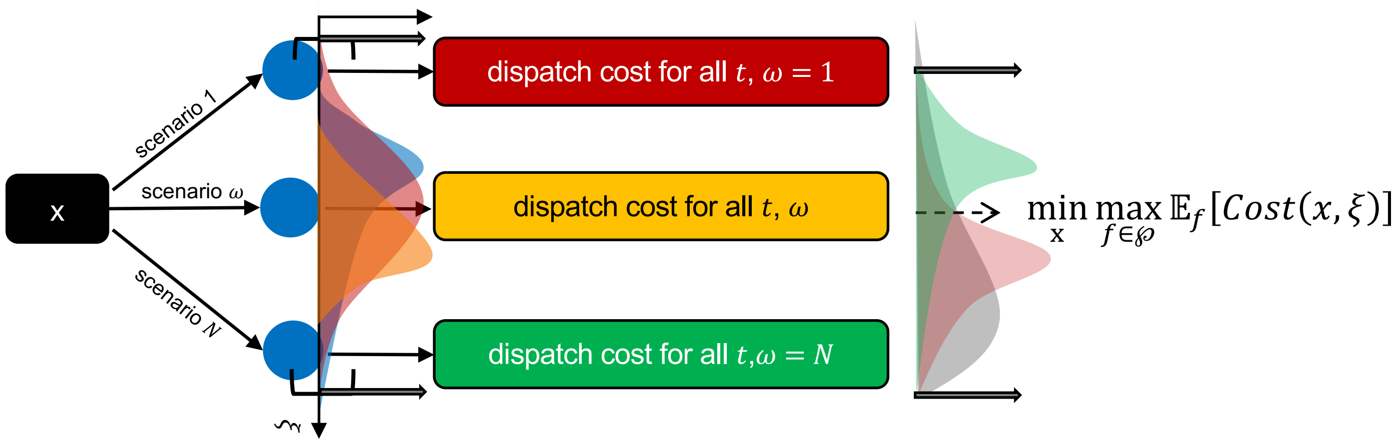


New models for short-term uncertainty

- Distributionally Robust

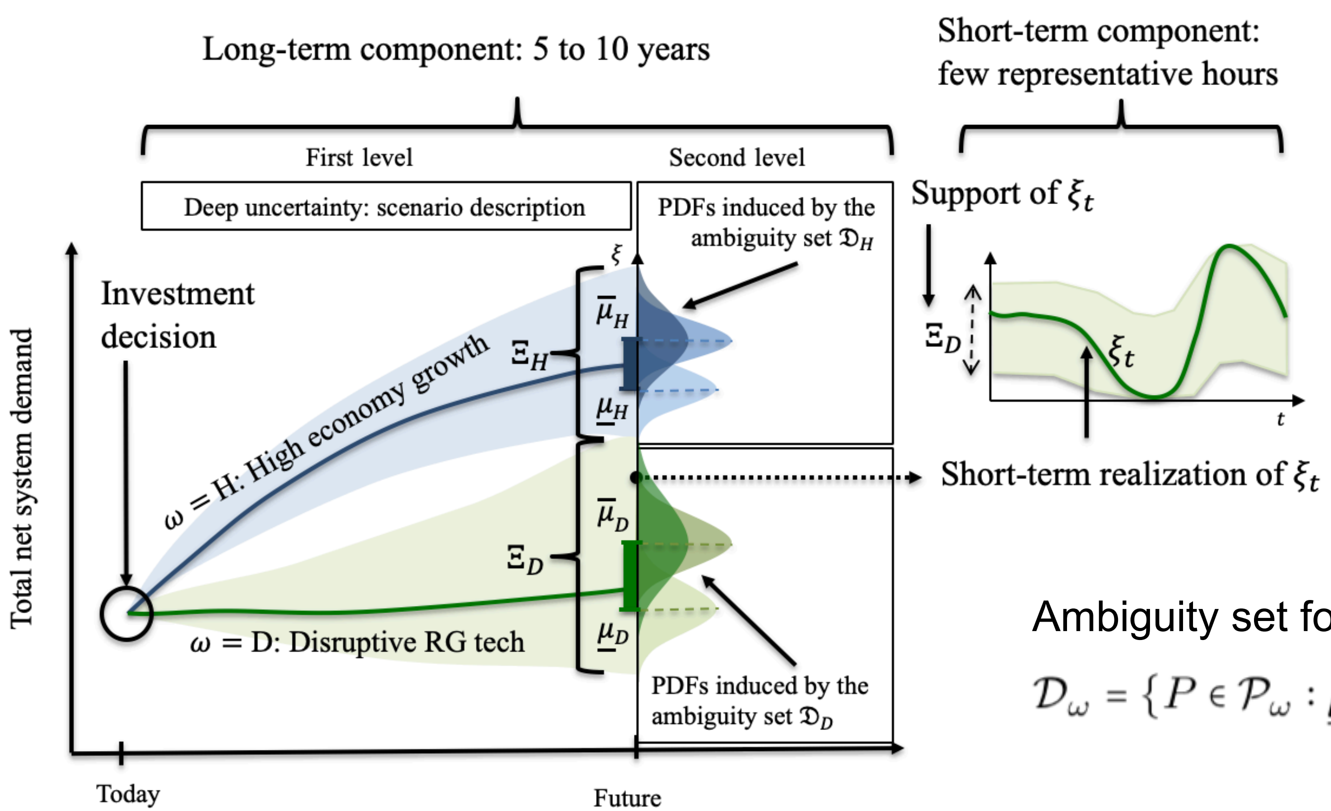


- First-stage decisions minimize the worst-case expected cost among all distributions considering all scenarios



Decision stages and information level

- Current industry practices based on long-term scenario studies define ranges for the expected net demand and its support for each $\omega \in \Omega$
- Conditional ambiguity sets are coupled to the long-term information



Ambiguity set for long-term scenario ω:

$$\mathcal{D}_\omega = \{ P \in \mathcal{P}_\omega : \underline{\boldsymbol{\mu}}_\omega \leq \mathbb{E}_P[\tilde{\boldsymbol{\xi}} \,|\, \mathcal{S}_\omega] \leq \overline{\boldsymbol{\mu}}_\omega \}$$

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Minimize the worst-case expected total cost

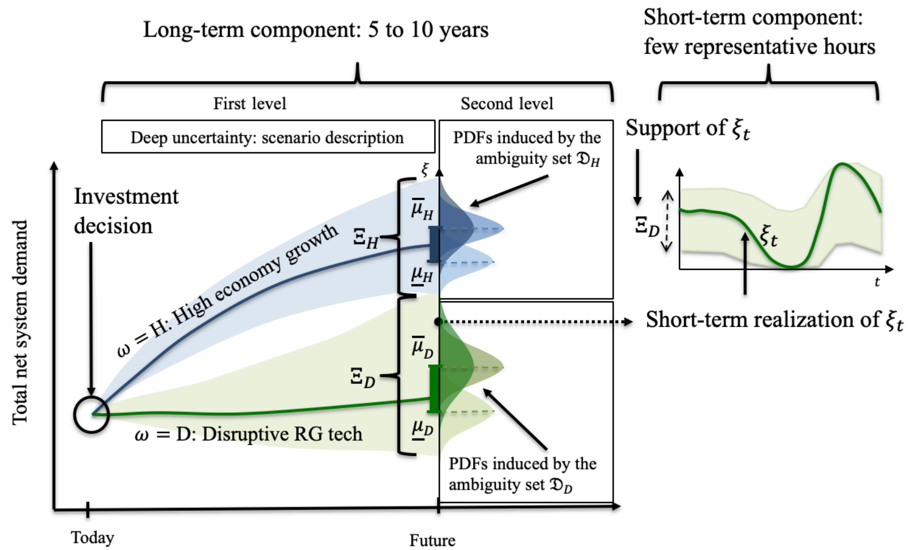
- A new multi-scale distributionally robust model for TEP based on the concept of multiple conditional ambiguity sets, which is a novelty in the literature of DRO, and suitable to the current industry practices.

Distributionally robust model

$$z^*_{DR} = \min_{\mathbf{x} \in \mathcal{X}} \left\{ \mathbf{c}_{inv}^\top \mathbf{x} + \sum_{\omega \in \Omega} \rho_\omega H_{DR}(x, \omega) \right\}$$

Distributionally robust recourse function

$$H_{DR}(\mathbf{x}, \omega) = \sup_{P \in \mathcal{D}_\omega} \mathbb{E}_P[g(\mathbf{x}, \tilde{\boldsymbol{\xi}}, \omega) | S_\omega]$$



The TEP model

Minimize the worst-case expected total cost

- A new multi-scale distributionally robust model for TEP based on the concept of multiple conditional ambiguity sets, which is a novelty in the literature of DRO, and suitable to the current industry practices.

Distributionally robust model

$$z^*_{DR} = \min_{\mathbf{x} \in \mathcal{X}} \left\{ \mathbf{c}_{inv}^{\top} \mathbf{x} + \sum_{\omega \in \Omega} \rho_\omega H_{DR}(x, \omega) \right\}$$

Distributionally robust recourse function

$$H_{DR}(\mathbf{x}, \omega) = \sup_{P \in \mathcal{D}_\omega} \mathbb{E}_P [g(\mathbf{x}, \tilde{\boldsymbol{\xi}}, \omega) | S_\omega]$$

- Operational model and its compact formulation

$$\begin{aligned} g(\mathbf{x}, \boldsymbol{\xi}, \omega) = & \min_{\substack{\mathbf{q}, \mathbf{f}, \theta \\ \phi_t^{(-)}, \phi_t^{(+)} \geq \mathbf{0}}} \sum_{t \in \mathcal{T}} (\mathbf{c}_\omega^{\top} \mathbf{q}_t + \lambda_\omega^{(-)\top} \phi_t^{(-)} + \lambda_\omega^{(+)\top} \phi_t^{(+)}) \\ & \text{s.t.:} \\ & \mathbf{G}\mathbf{q}_t + \mathbf{A}\mathbf{f}_t = \mathbf{B}_{t,\omega} \boldsymbol{\xi}_t - \phi_t^{(-)} + \phi_t^{(+)}, \ \forall t \in \mathcal{T} \\ & |\mathbf{f}_t - \mathbf{S}\theta_t| \leq \mathbf{C}(\mathbf{e} - \mathbf{x}), \ \forall t \in \mathcal{T} \\ & -\overline{\mathbf{F}}\mathbf{x} - \overline{\mathbf{f}} \leq \mathbf{f}_t \leq \overline{\mathbf{f}} + \overline{\mathbf{F}}\mathbf{x}, \ \forall t \in \mathcal{T} \\ & -\mathbf{r}_\omega^{dw} \leq \mathbf{q}_t - \mathbf{q}_{t-1} \leq \mathbf{r}_\omega^{up}, \ \forall t \in \mathcal{T} \\ & \mathbf{0} \leq \mathbf{q}_t \leq \overline{\mathbf{q}}_\omega, \ \forall t \in \mathcal{T}. \end{aligned}$$

Convex on $\boldsymbol{x}$ and $\boldsymbol{\xi}$

$$g(\mathbf{x}, \boldsymbol{\xi}, \omega) = \min_{\mathbf{y} \geq 0} \left\{ \mathbf{h}_\omega^{\top} \mathbf{y} \,\middle|\, \mathbf{W}_\omega \mathbf{y} \geq \mathbf{b}_\omega + \mathbf{B}_\omega \boldsymbol{\xi} - \mathbf{T}_\omega \mathbf{x} \right\}$$

Form the infinite to finite formulation

The worst-case recourse function is a semi-infinite problem

- Primal formulation

$$
\begin{aligned}
H_{DR}(\mathbf{x},\omega) = \sup_{P \in \mathcal{P}_\omega} & \int_{\mathcal{S}_\omega} g(\mathbf{x},\tilde{\boldsymbol{\xi}},\omega)\,dP \\
\text{s.t.:} \quad & \int_{\mathcal{S}_\omega} dP = 1 && : \alpha_{0\omega} \\
& \underline{\boldsymbol{\mu}}_\omega \le \int_{\mathcal{S}_\omega} \tilde{\boldsymbol{\xi}}\,dP \le \overline{\boldsymbol{\mu}}_\omega && : \underline{\boldsymbol{\alpha}}_\omega,\ \overline{\boldsymbol{\alpha}}_\omega
\end{aligned}
$$

- Dual formulation

$$
\begin{aligned}
H_{DR}(\mathbf{x},\omega) = & \min_{\alpha_0,\overline{\boldsymbol{\alpha}},\underline{\boldsymbol{\alpha}}} \left\{ \alpha_{0\omega} + \overline{\boldsymbol{\mu}}_\omega^\top \overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\mu}}_\omega^\top \underline{\boldsymbol{\alpha}}_\omega \right\} \\
\text{s.t.:} \quad & \alpha_{0\omega} + (\overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\alpha}}_\omega)^\top \tilde{\boldsymbol{\xi}}(s) \ge g(\mathbf{x},\tilde{\boldsymbol{\xi}}(s),\omega),\ \forall s \in \mathcal{S}_\omega
\end{aligned}
$$

Dual constraints are always bind in extreme points

- Equivalent dual formulation

$$H_{DR}(\mathbf{x},\omega) = \min_{\alpha_0,\overline{\boldsymbol{\alpha}},\underline{\boldsymbol{\alpha}}} \left\{ \alpha_{0\omega} + \overline{\boldsymbol{\mu}}_\omega^\top \overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\mu}}_\omega^\top \underline{\boldsymbol{\alpha}}_\omega \right\}$$

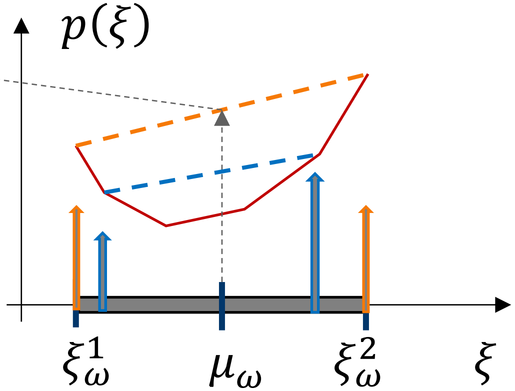
$$\text{s.t.:} \quad \alpha_{0\omega} + (\overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\alpha}}_\omega)^\top \boldsymbol{\xi}_\omega^k \geq g(\mathbf{x}, \boldsymbol{\xi}_\omega^k, \omega), \ \forall k \in K_\omega$$

- Equivalent primal formulation

$$H_{DR}(\mathbf{x},\omega) = \max_{p^k} \sum_{k \in K_\omega} g(\mathbf{x}, \boldsymbol{\xi}_\omega^k, \omega)\, p^k$$

$$\text{s.t.:} \quad \sum_{k \in K_\omega} p^k = 1$$

$$\underline{\boldsymbol{\mu}}_\omega \leq \sum_{k \in K_\omega} \boldsymbol{\xi}_\omega^k p^k \leq \overline{\boldsymbol{\mu}}_\omega,$$



The MILP DRO TEP model

- Extended equivalent dual formulation (explicit operation model)

$$H_{DR}(\mathbf{x},\omega) = \min_{\alpha_0,\overline{\boldsymbol{\alpha}},\underline{\boldsymbol{\alpha}}} \left\{ \alpha_{0\omega} + \overline{\boldsymbol{\mu}}_\omega^\top \overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\mu}}_\omega^\top \underline{\boldsymbol{\alpha}}_\omega \right\}$$

$$\text{s.t.:} \quad \alpha_{0\omega} + (\overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\alpha}}_\omega)^\top \boldsymbol{\xi}_\omega^k \geq \mathbf{h}_\omega^\top \mathbf{y}_\omega^k, \ \forall k \in K_\omega$$

$$\mathbf{W}_\omega \mathbf{y}_\omega^k \geq \mathbf{b}_\omega + \mathbf{B}_\omega \boldsymbol{\xi}_\omega^k - \mathbf{T}_\omega \mathbf{x}, \forall k \in K_\omega.$$

- Equivalent MILP DRO TEP

$$z_{DR}^* = \min_{\substack{\mathbf{x},\mathbf{y}_\omega^k,\alpha_{0\omega}, \\ \overline{\boldsymbol{\alpha}}_\omega,\underline{\boldsymbol{\alpha}}_\omega}} \mathbf{c}_{inv}^\top \mathbf{x} + \sum_{\omega\in\Omega} \rho_\omega \left[\alpha_{0\omega} + \overline{\boldsymbol{\mu}}_\omega^\top \overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\mu}}_\omega^\top \underline{\boldsymbol{\alpha}}_\omega \right]$$

$$\text{s.t.:} \quad \mathbf{x} \in \mathcal{X}$$

$$\mathbf{h}_\omega^\top \mathbf{y}_\omega^k \leq \alpha_{0\omega} + (\overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\alpha}}_\omega)^\top \boldsymbol{\xi}_\omega^k, \forall \omega \in \Omega, k \in K_\omega$$

$$\mathbf{W}_\omega \mathbf{y}_\omega^k \geq \mathbf{b}_\omega + \mathbf{B}_\omega \boldsymbol{\xi}_\omega^k - \mathbf{T}_\omega \mathbf{x}, \forall \omega \in \Omega, k \in K_\omega.$$

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Solution Methodology: in a given iteration j

- Master problem defines $(LB^{(j)}, \mathbf{x}^{(j)}) \leftarrow$

$$\left\{ \begin{array}{l} \min\limits_{\substack{\mathbf{x},\mathbf{y}_\omega^k,\alpha_{0\omega},\\ \overline{\boldsymbol{\alpha}}_\omega,\underline{\boldsymbol{\alpha}}_\omega}} \ \mathbf{c}_{inv}^\top \mathbf{x} + \sum\limits_{\omega\in\Omega} \rho_\omega \left[\alpha_{0\omega} + \overline{\boldsymbol{\mu}}_\omega^\top \overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\mu}}_\omega^\top \underline{\boldsymbol{\alpha}}_\omega\right] \\ \text{s.t.: } \mathbf{x} \in \mathcal{X} \\ \qquad \alpha_{0\omega} + (\overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\alpha}}_\omega)^\top \boldsymbol{\xi}_\omega^k \geq \mathbf{h}_\omega^\top \mathbf{y}_\omega^k, \forall \omega \in \Omega, k \in K_\omega^{(j)} \\ \qquad \mathbf{W}_\omega \mathbf{y}_\omega^k \geq \mathbf{b}_\omega + \mathbf{B}_\omega \boldsymbol{\xi}_\omega^k - \mathbf{T}_\omega \mathbf{x}, \forall \omega \in \Omega, k \in K_\omega^{(j)}. \end{array} \right.$$

$$K_\omega^{(j)} = \{0, 1, \ldots, |\mathcal{E}_\omega^{(j)}|\}$$

$$\underline{H}_{DR}(\mathbf{x}^{(j)}, \omega) = \max_{p^k} \sum_{k\in K_\omega^{(j)}} g(\mathbf{x}^{(j)}, \boldsymbol{\xi}_\omega^k, \omega)\, p^k$$

$$\text{s.t.: } \sum_{k\in K_\omega^{(j)}} p^k = 1 \qquad : \alpha_{0\omega}^{(j)}$$

$$\underline{\boldsymbol{\mu}}_\omega \leq \sum_{k\in K_\omega^{(j)}} \boldsymbol{\xi}_\omega^k p^k \leq \overline{\boldsymbol{\mu}}_\omega \qquad : \underline{\boldsymbol{\alpha}}_\omega^{(j)}, \overline{\boldsymbol{\alpha}}_\omega^{(j)}$$

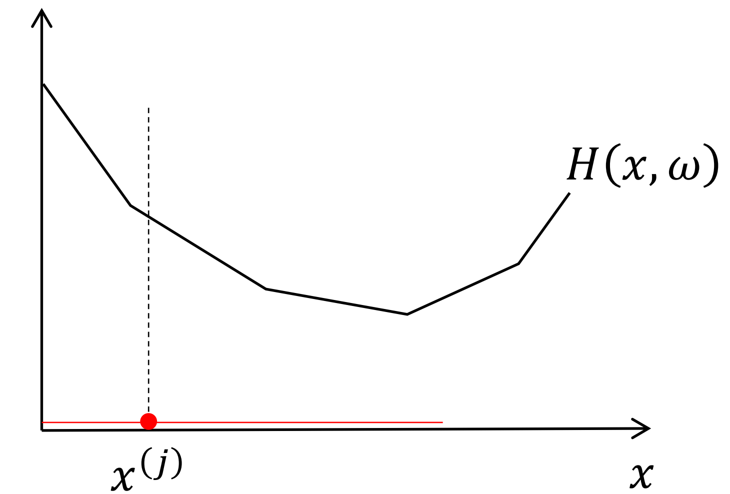
- Subproblem updates $\mathcal{E}_\omega^{(j+1)} \leftarrow \mathcal{E}_\omega^{(j)}$

$$\mathcal{E}_\omega^{(j)} \leftarrow \mathcal{E}_\omega^{(j)} \cup \{\boldsymbol{\xi}_\omega^*\}$$

$$(\overline{c}_\omega^*, \boldsymbol{\xi}_\omega^*) \leftarrow \max_{\boldsymbol{\xi}\in\Xi_\omega} \left\{ g(\mathbf{x}^{(j)}, \boldsymbol{\xi}, \omega) - \alpha_{0\omega}^{(j)} - \left(\overline{\boldsymbol{\alpha}}_\omega^{(j)} - \underline{\boldsymbol{\alpha}}_\omega^{(j)}\right)^\top \boldsymbol{\xi} \right\}$$

Solution Methodology

- Master problem defines a trial investment plan $x^{(j)}$ that should be evaluated in the recourse function
- A lower bound for the problem is defined

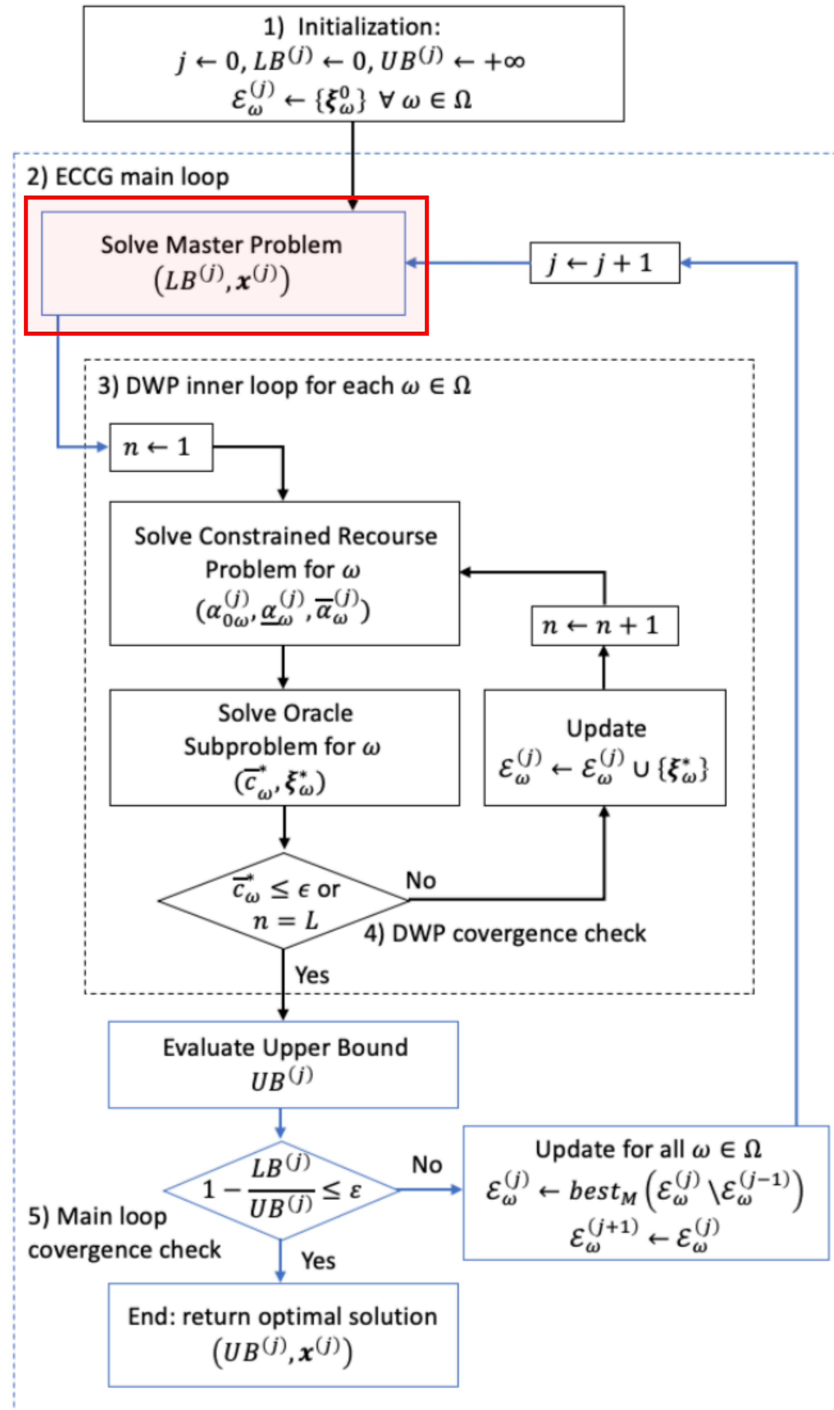


$$(LB^{(j)}, \mathbf{x}^{(j)}) \leftarrow \min_{\substack{\mathbf{x}, \mathbf{y}^k_\omega, \alpha_{0\omega}, \\ \overline{\boldsymbol{\alpha}}_\omega, \underline{\boldsymbol{\alpha}}_\omega}} \mathbf{c}_{inv}^\top \mathbf{x} + \sum_{\omega \in \Omega} \rho_\omega \left[\alpha_{0\omega} + \overline{\boldsymbol{\mu}}_\omega^\top \overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\mu}}_\omega^\top \underline{\boldsymbol{\alpha}}_\omega \right]$$

$$\text{s.t.: } \mathbf{x} \in \mathcal{X}$$

$$\alpha_{0\omega} + (\overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\alpha}}_\omega)^\top \boldsymbol{\xi}^k_\omega \geq \mathbf{h}_\omega^\top \mathbf{y}^k_\omega, \forall \omega \in \Omega, k \in K^{(j)}_\omega$$

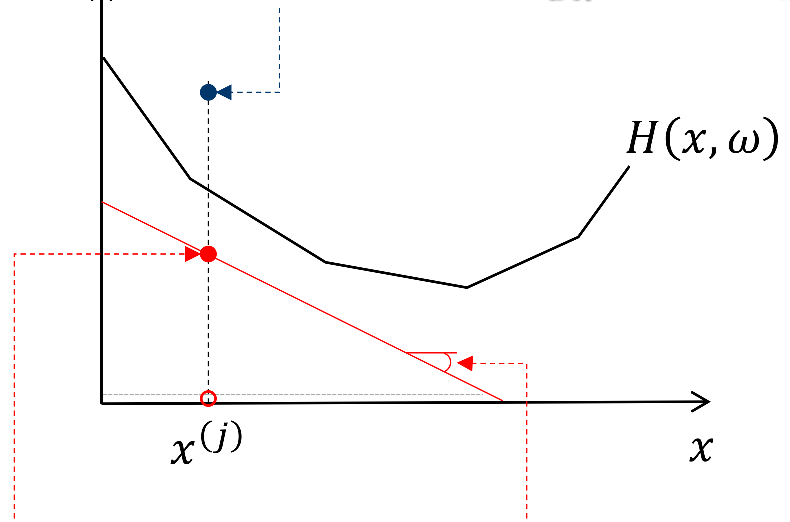
$$\mathbf{W}_\omega \mathbf{y}^k_\omega \geq \mathbf{b}_\omega + \mathbf{B}_\omega \boldsymbol{\xi}^k_\omega - \mathbf{T}_\omega \mathbf{x}, \forall \omega \in \Omega, k \in K^{(j)}_\omega.$$



Dantzig Wolfe Procedure

- The inner loop provides a LB and Dantzig Wolfe-like UB for the recourse function based on the reduced cost

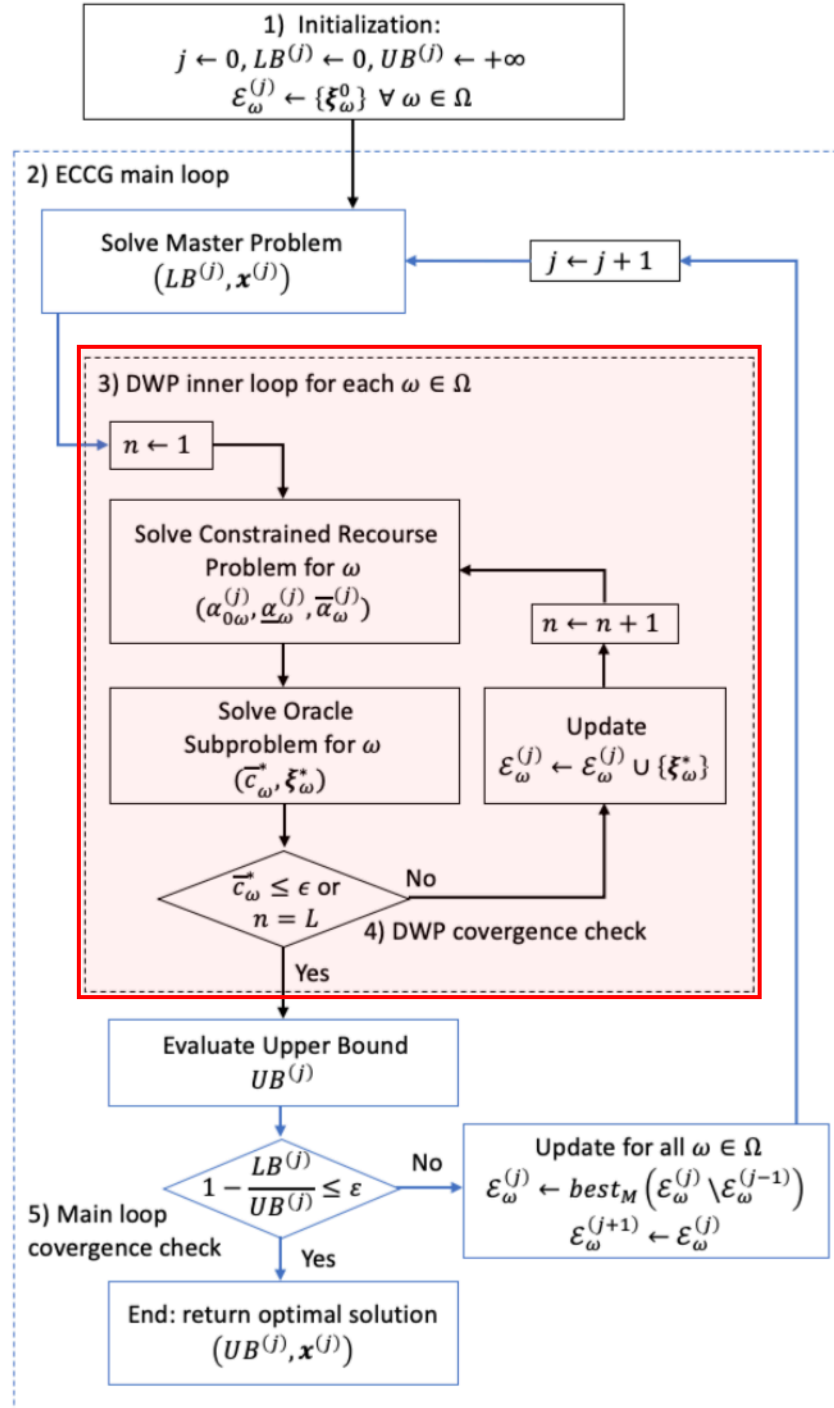
$$\overline{H}_{DR}(\mathbf{x}^{(j)},\omega) = \underline{H}_{DR}(\mathbf{x}^{(j)},\omega) + \overline{c}^*_\omega.$$



$$\underline{H}_{DR}(\mathbf{x}^{(j)},\omega) = \max_{p^k} \sum_{k\in K^{(j)}_\omega} g(\mathbf{x}^{(j)},\xi^k_\omega,\omega)\,p^k$$

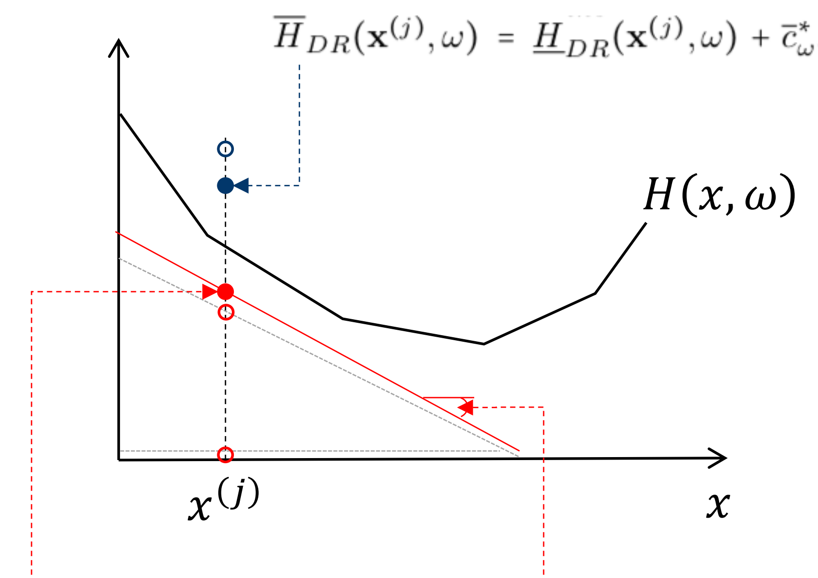
$$\text{s.t.:} \sum_{k\in K^{(j)}_\omega} p^k = 1 \qquad : \alpha^{(j)}_{0\omega}$$

$$\underline{\mu}_\omega \le \sum_{k\in K^{(j)}_\omega} \xi^k_\omega p^k \le \overline{\mu}_\omega \qquad : \underline{\alpha}^{(j)}_\omega, \overline{\alpha}^{(j)}_\omega$$



Dantzig Wolfe Procedure

- The inner loop enhances the LB and UB recourse function approximation at $x^{(j)}$

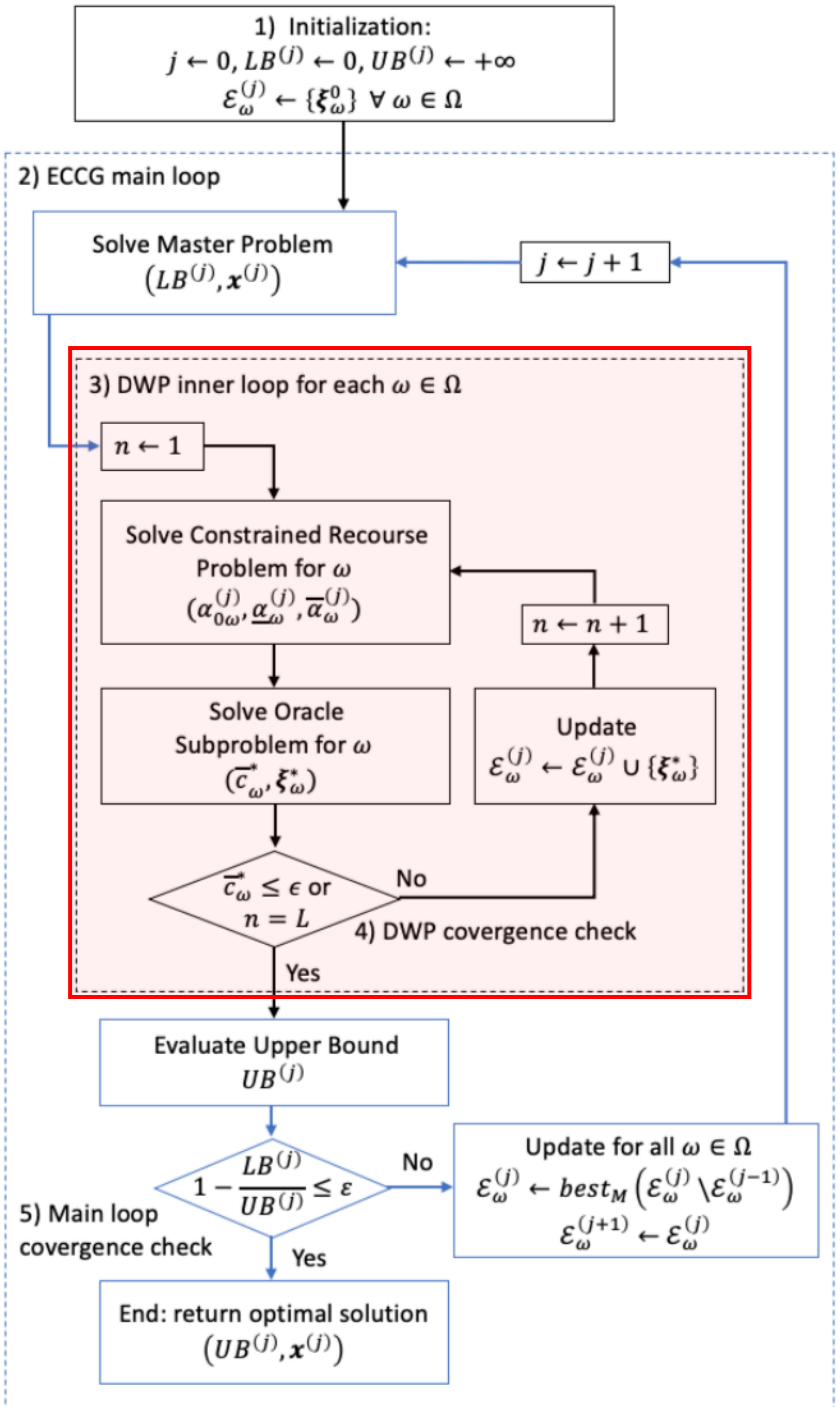


$$\underline{H}_{DR}(\mathbf{x}^{(j)},\omega) = \max_{p^k} \sum_{k\in K^{(j)}_\omega} g(\mathbf{x}^{(j)},\boldsymbol{\xi}^k_\omega,\omega)\,p^k$$

$$\text{s.t.:} \sum_{k\in K^{(j)}_\omega} p^k = 1 \qquad : \alpha^{(j)}_{0\omega}$$

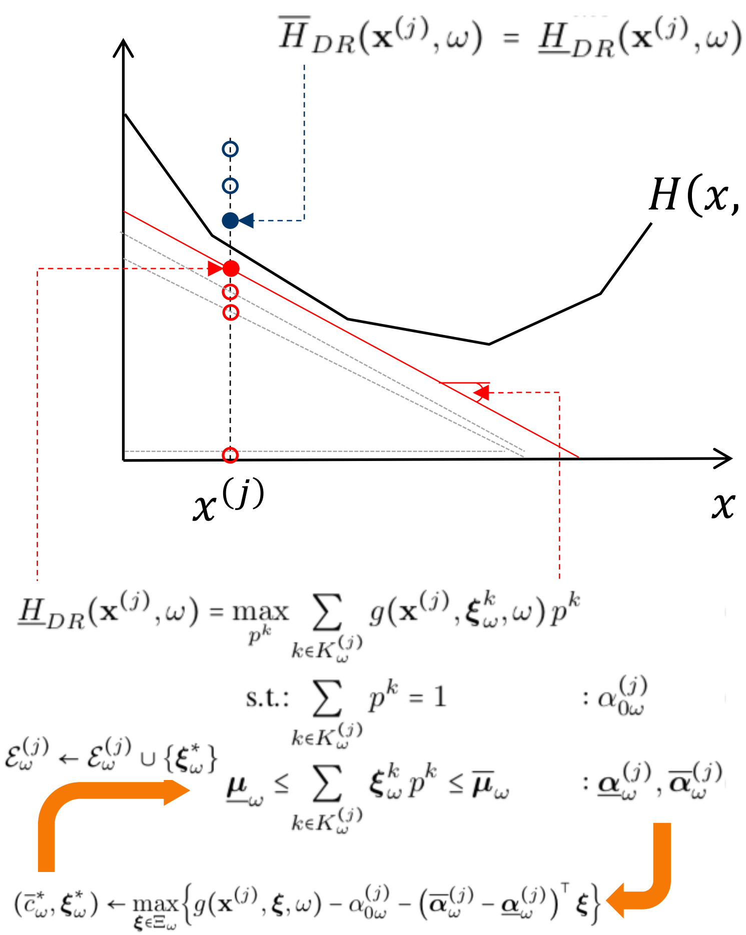
$$\mathcal{E}^{(j)}_\omega \leftarrow \mathcal{E}^{(j)}_\omega \cup \{\boldsymbol{\xi}^*_\omega\} \qquad \underline{\boldsymbol{\mu}}_\omega \le \sum_{k\in K^{(j)}_\omega} \boldsymbol{\xi}^k_\omega p^k \le \overline{\boldsymbol{\mu}}_\omega \qquad : \underline{\boldsymbol{\alpha}}^{(j)}_\omega, \overline{\boldsymbol{\alpha}}^{(j)}_\omega$$

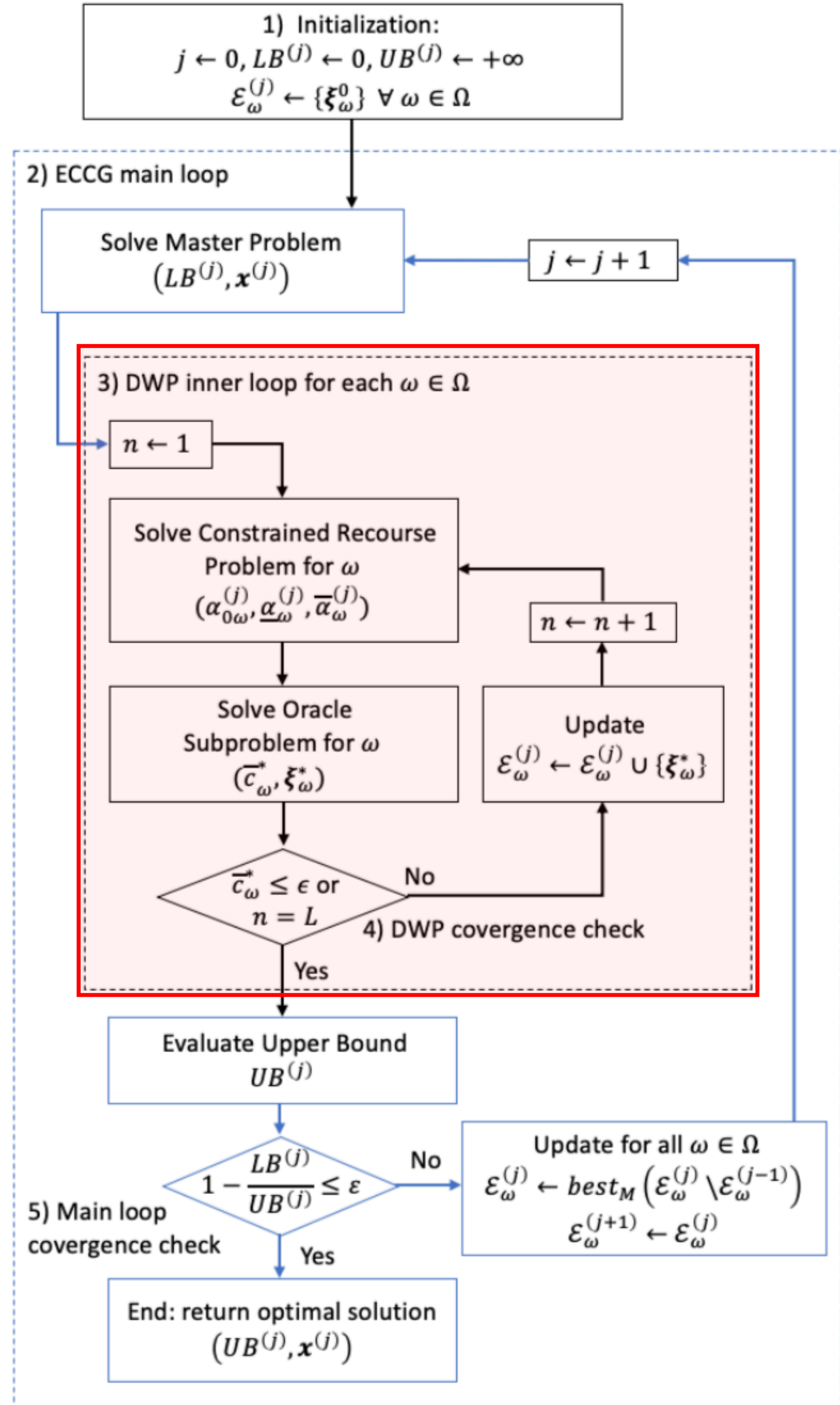
$$(\bar{c}^*_\omega, \boldsymbol{\xi}^*_\omega) \leftarrow \max_{\boldsymbol{\xi}\in\Xi_\omega}\left\{g(\mathbf{x}^{(j)},\boldsymbol{\xi},\omega) - \alpha^{(j)}_{0\omega} - \left(\overline{\boldsymbol{\alpha}}^{(j)}_\omega - \underline{\boldsymbol{\alpha}}^{(j)}_\omega\right)^\top \boldsymbol{\xi}\right\}$$



Dantzig Wolfe Procedure

- The inner loop enhances the LB and UB recourse function approximation at $x^{(j)}$

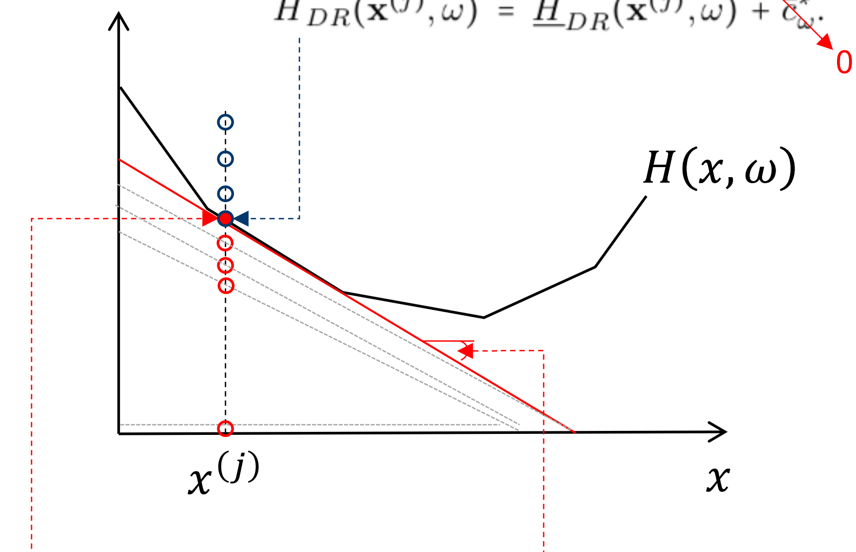




Dantzig Wolfe Procedure

- When the DWP converges: reduced cost is zero and the recourse function is perfectly approximated at $x^{(j)}$

$$\overline{H}_{DR}(\mathbf{x}^{(j)},\omega) = \underline{H}_{DR}(\mathbf{x}^{(j)},\omega) + \bar{c}^*_\omega \rightarrow 0$$



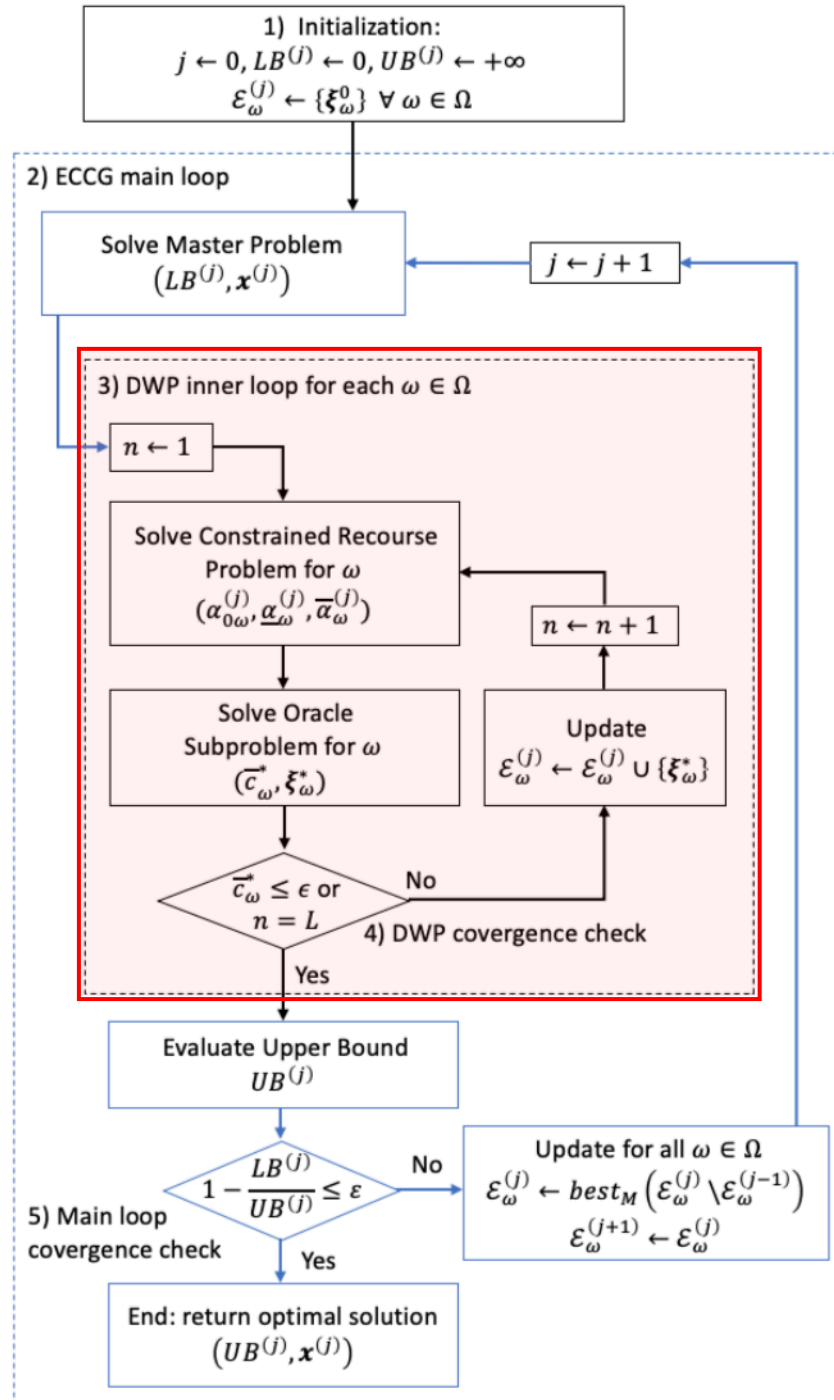
$$\underline{H}_{DR}(\mathbf{x}^{(j)},\omega) = \max_{p^k} \sum_{k\in K_\omega^{(j)}} g(\mathbf{x}^{(j)},\boldsymbol{\xi}^k_\omega,\omega)\,p^k$$

$$\text{s.t.: } \sum_{k\in K_\omega^{(j)}} p^k = 1 \qquad : \alpha_{0\omega}^{(j)}$$

$$\underline{\boldsymbol{\mu}}_\omega \le \sum_{k\in K_\omega^{(j)}} \boldsymbol{\xi}^k_\omega p^k \le \overline{\boldsymbol{\mu}}_\omega \qquad : \underline{\boldsymbol{\alpha}}_\omega^{(j)}, \overline{\boldsymbol{\alpha}}_\omega^{(j)}$$

$$\mathcal{E}_\omega^{(j)} \leftarrow \mathcal{E}_\omega^{(j)} \cup \{\boldsymbol{\xi}^*_\omega\}$$

$$(\bar{c}^*_\omega, \boldsymbol{\xi}^*_\omega) \leftarrow \max_{\boldsymbol{\xi}\in\Xi_\omega}\left\{g(\mathbf{x}^{(j)},\boldsymbol{\xi},\omega) - \alpha_{0\omega}^{(j)} - \left(\overline{\boldsymbol{\alpha}}_\omega^{(j)} - \underline{\boldsymbol{\alpha}}_\omega^{(j)}\right)^\top \boldsymbol{\xi}\right\}$$



Dantzig Wolfe Procedure

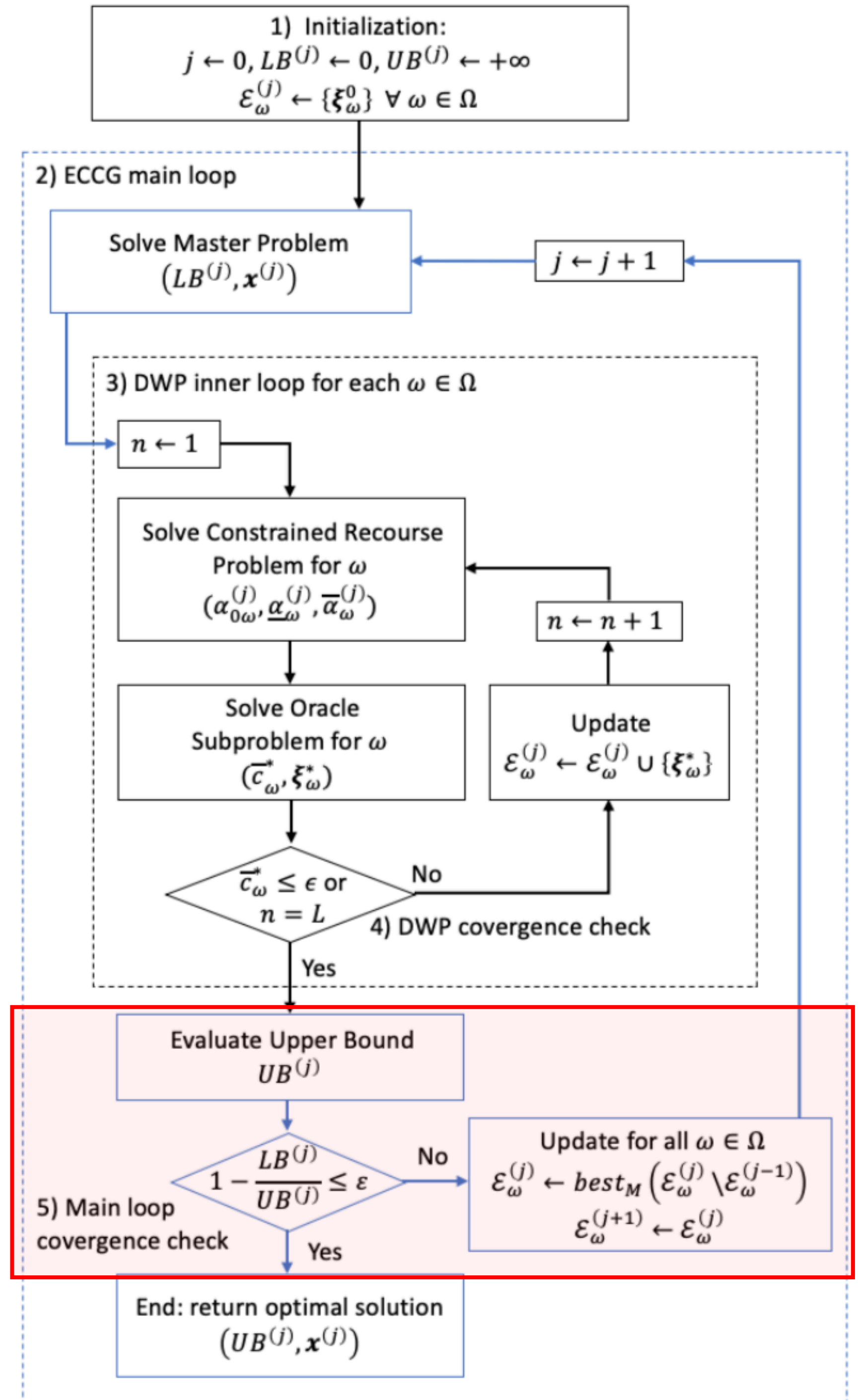
- A tighter Upper Bound for the problem is found based on the recourse function UB ($\overline{H}$)

$$UB^{(j)} = \mathbf{c}_{inv}^{\top}\mathbf{x}^{(j)} + \sum_{\omega\in\Omega} \rho_\omega \overline{H}_{DR}(\mathbf{x}^{(j)}, \omega)$$

Theorem 1 (ECCG algorithm upper bound)

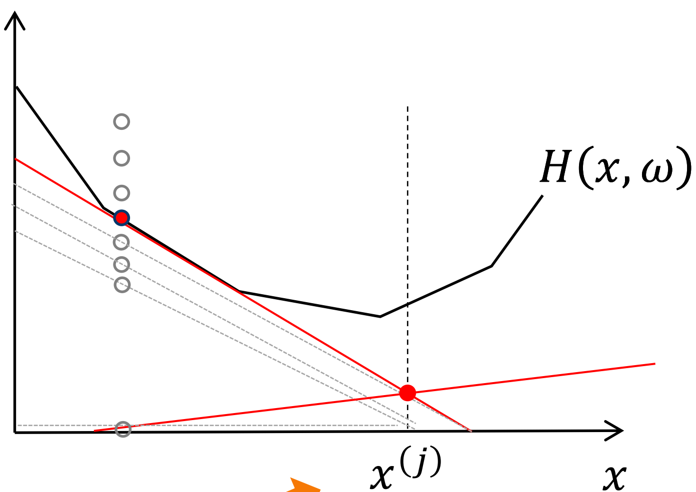
1) $UB^{(j)} = \mathbf{c}_{inv}^{\top}\mathbf{x}^{(j)} + \sum_{\omega\in\Omega} \rho_\omega \overline{H}_{DR}(\mathbf{x}^{(j)}, \omega)$ *is an upper bound for the DRO problem* (11), *i.e.,* $z_{DR}^* \leq \mathbf{c}_{inv}^{\top}\mathbf{x}^{(j)} + \sum_{\omega\in\Omega} \rho_\omega \overline{H}_{DR}(\mathbf{x}^{(j)}, \omega)$.
2) $UB^{(j)}$ *is tight for* $\mathbf{x}^{(j)}$ *when* $\epsilon = 0$ *and* $L = +\infty$, *i.e.,* $UB^{(j)} = \mathbf{c}_{inv}^{\top}\mathbf{x}^{(j)} + H_{DR}(\mathbf{x}^{(j)}, \omega)$ *if the DWP converges with zero gap.*

- The best M scenarios are added into the master problem for the next iteration $j + 1$



Enhanced CCG algorithm

- In the next iteration ($j \leftarrow j+1$): the master problem finds another point and lower bound with M newly added scenarios

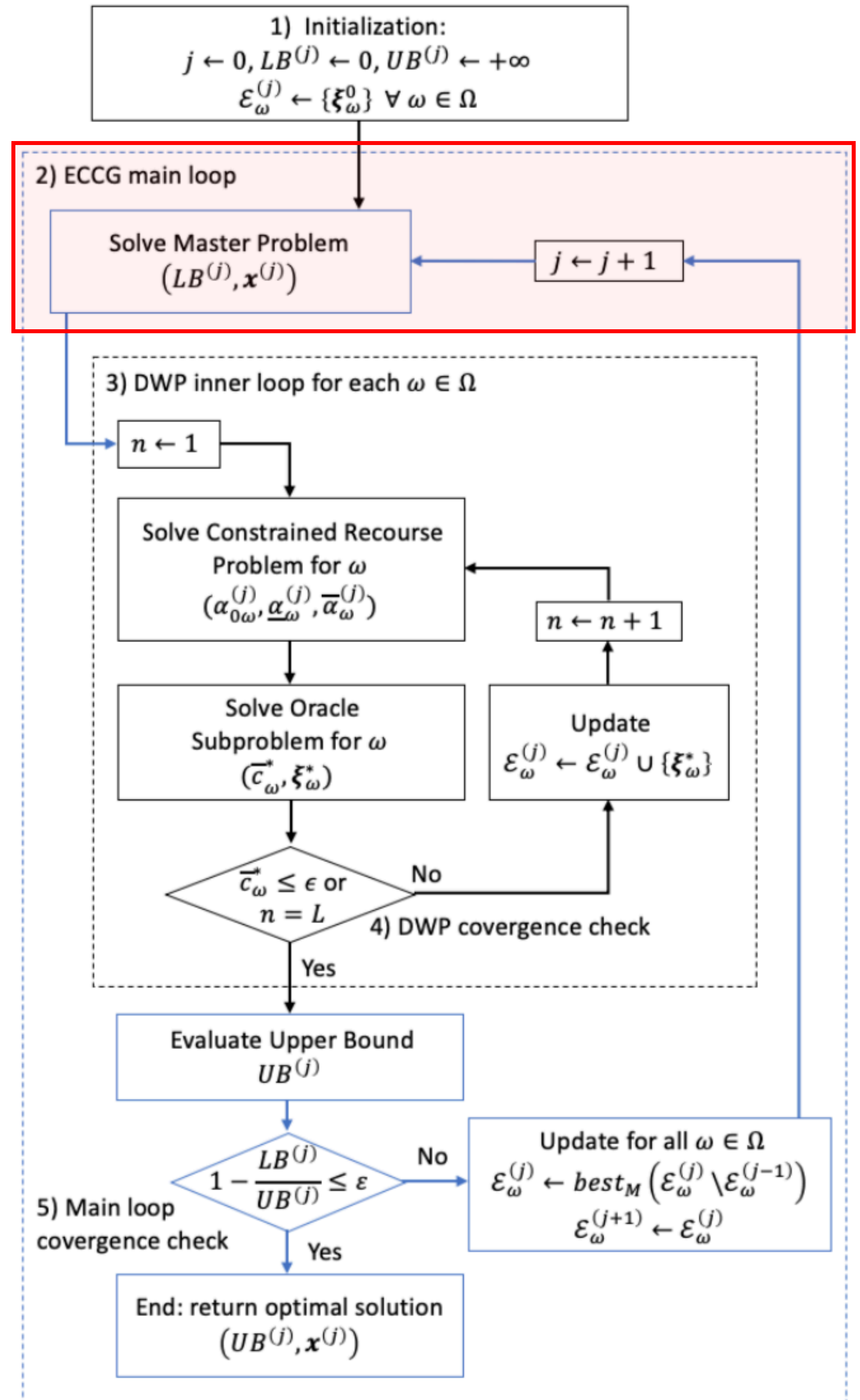


$$(LB^{(j)}, \mathbf{x}^{(j)}) \leftarrow \min_{\substack{\mathbf{x}, \mathbf{y}_\omega^k, \alpha_{0\omega}, \\ \overline{\boldsymbol{\alpha}}_\omega, \underline{\boldsymbol{\alpha}}_\omega}} \mathbf{c}_{inv}^\top \mathbf{x} + \sum_{\omega \in \Omega} \rho_\omega \left[\alpha_{0\omega} + \overline{\boldsymbol{\mu}}_\omega^\top \overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\mu}}_\omega^\top \underline{\boldsymbol{\alpha}}_\omega \right]$$

$$\text{s.t.: } \mathbf{x} \in \mathcal{X}$$

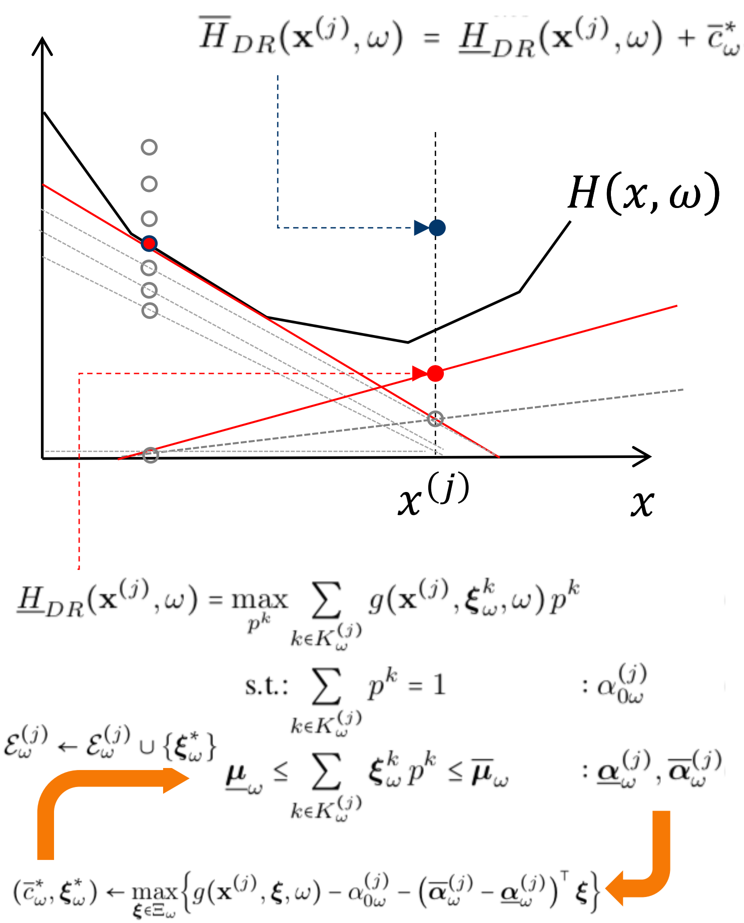
$$\alpha_{0\omega} + (\overline{\boldsymbol{\alpha}}_\omega - \underline{\boldsymbol{\alpha}}_\omega)^\top \boldsymbol{\xi}_\omega^k \geq \mathbf{h}_\omega^\top \mathbf{y}_\omega^k, \forall \omega \in \Omega, k \in K_\omega^{(j)}$$

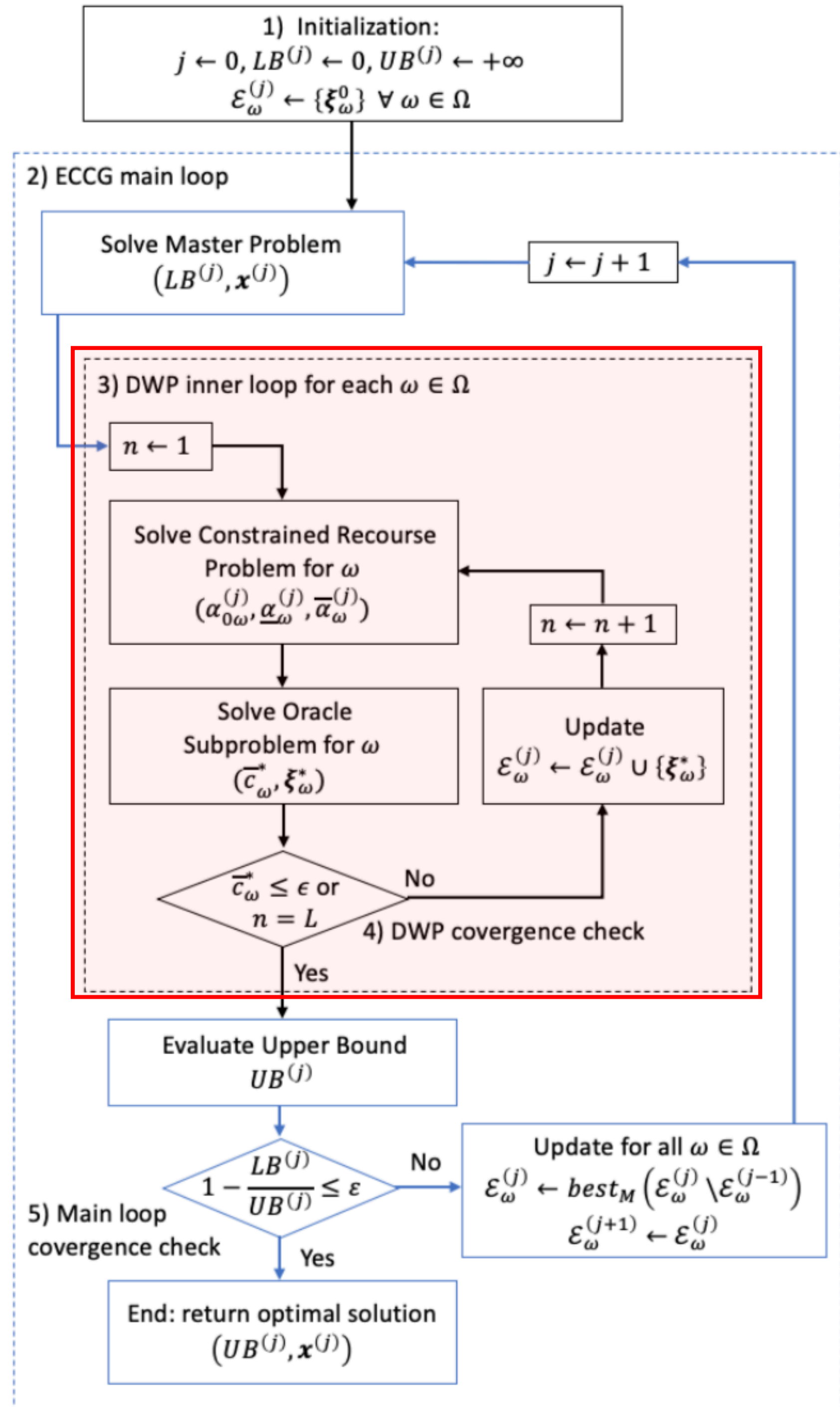
$$\mathbf{W}_\omega \mathbf{y}_\omega^k \geq \mathbf{b}_\omega + \mathbf{B}_\omega \boldsymbol{\xi}_\omega^k - \mathbf{T}_\omega \mathbf{x}, \forall \omega \in \Omega, k \in K_\omega^{(j)}.$$



Dantzig Wolfe Procedure

- The inner loop enhances the LB and UB recourse function approximation at $x^{(j)}$

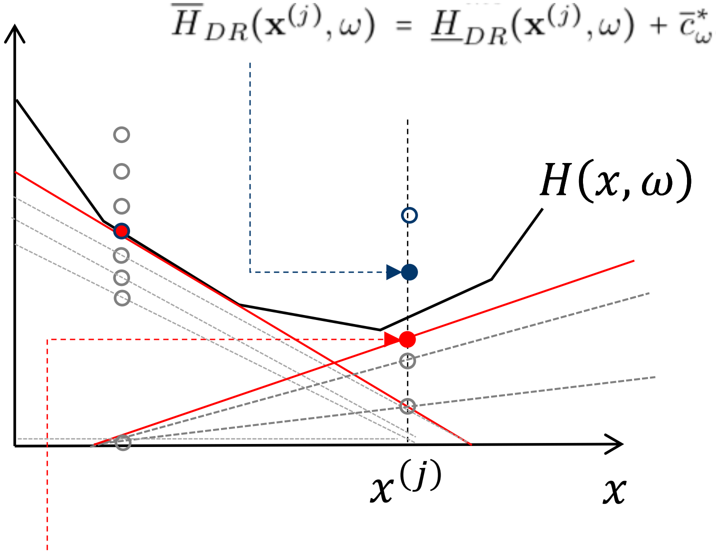




Dantzig Wolfe Procedure

- The inner loop enhances the LB and UB recourse function approximation at $x^{(j)}$

$$\overline{H}_{DR}(\mathbf{x}^{(j)},\omega) = \underline{H}_{DR}(\mathbf{x}^{(j)},\omega) + \bar{c}^*_\omega.$$



$$\underline{H}_{DR}(\mathbf{x}^{(j)},\omega) = \max_{p^k} \sum_{k\in K^{(j)}_\omega} g(\mathbf{x}^{(j)},\boldsymbol{\xi}^k_\omega,\omega)\,p^k$$

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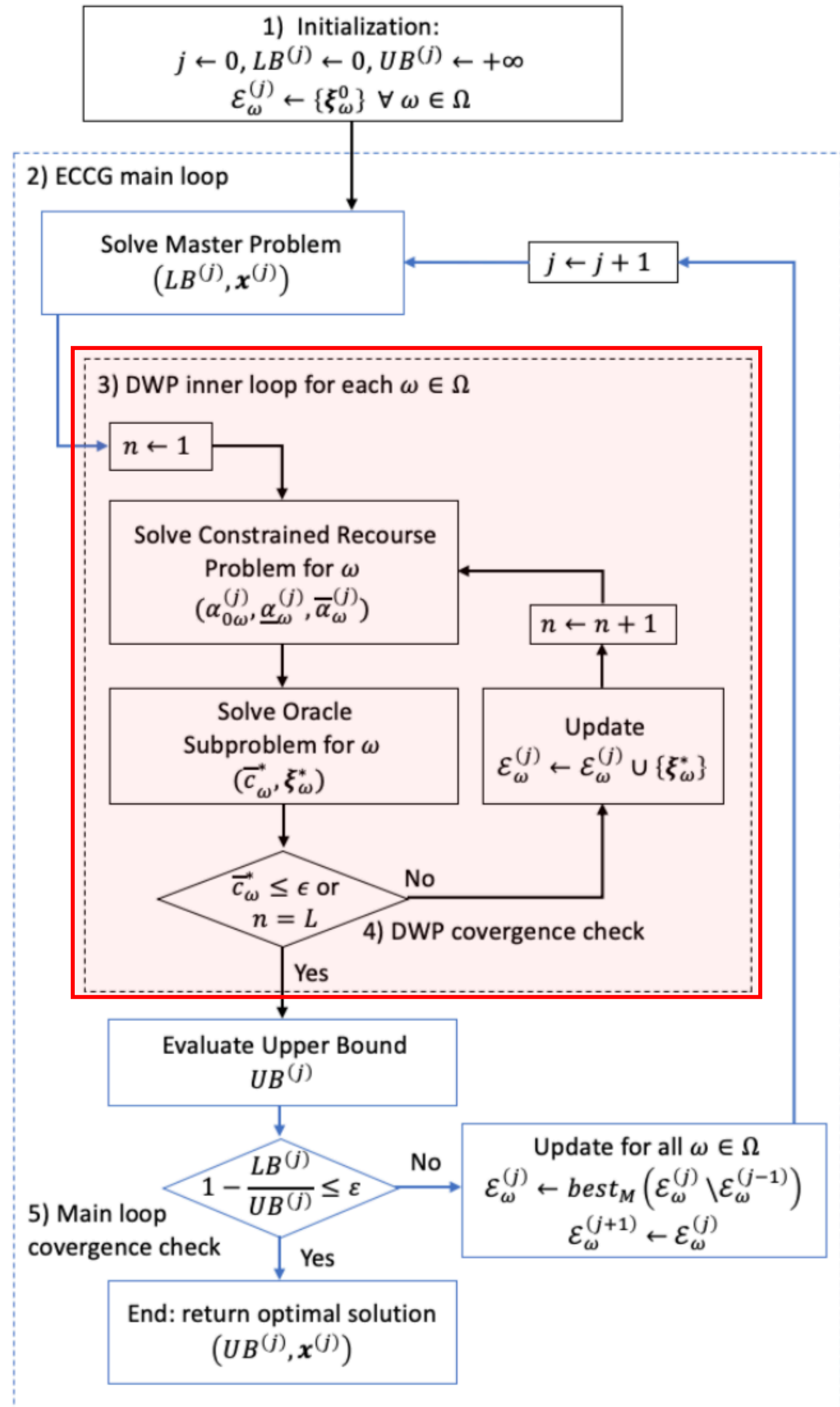
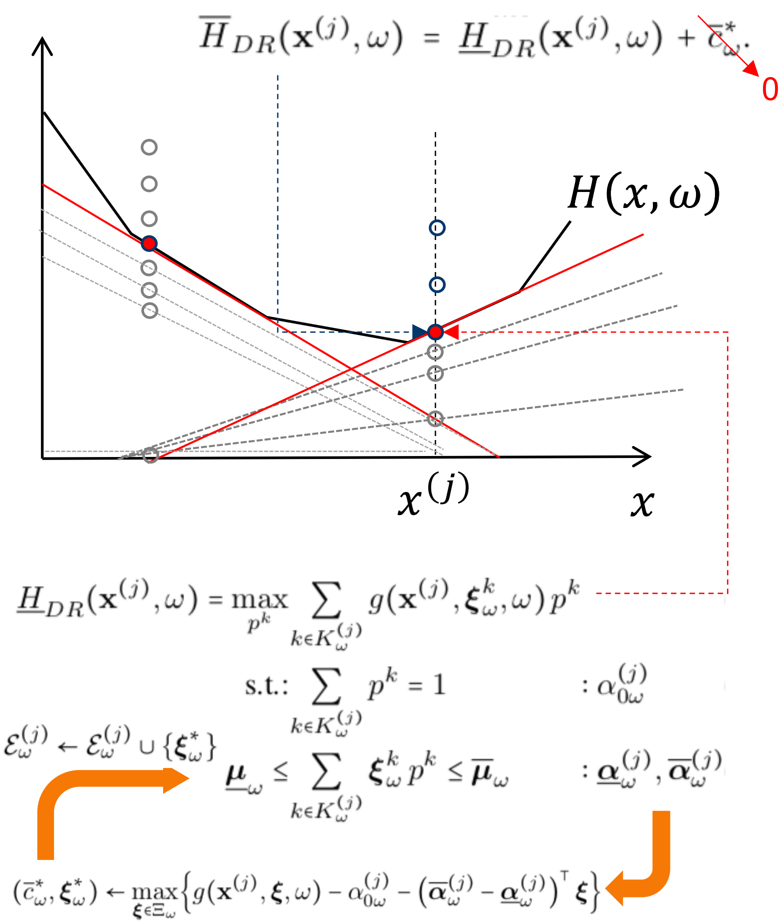
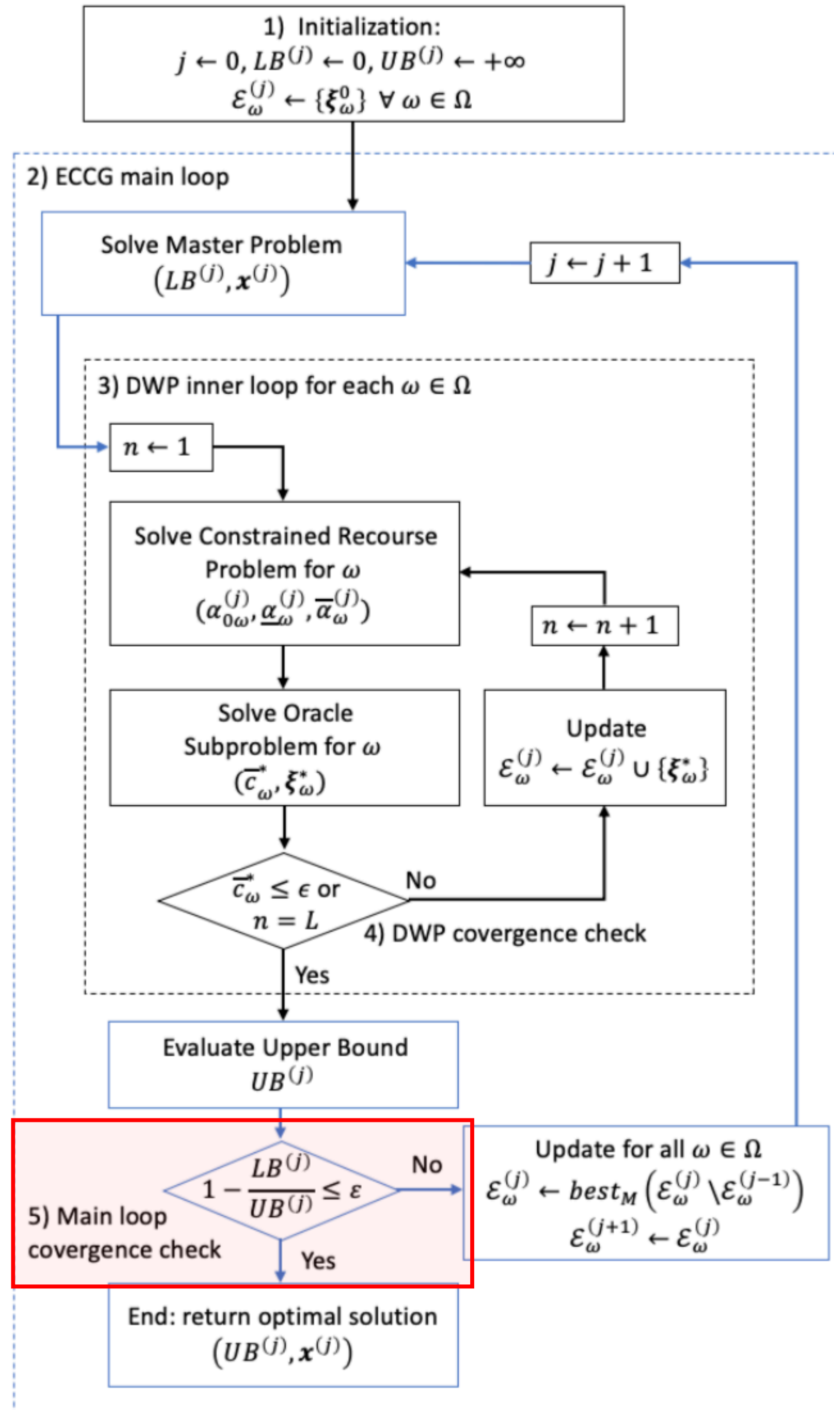
$$(\bar{c}^*_\omega, \boldsymbol{\xi}^*_\omega) \leftarrow \max_{\boldsymbol{\xi}\in\Xi_\omega}\left\{g(\mathbf{x}^{(j)},\boldsymbol{\xi},\omega) - \alpha^{(j)}_{0\omega} - \left(\overline{\boldsymbol{\alpha}}^{(j)}_\omega - \underline{\boldsymbol{\alpha}}^{(j)}_\omega\right)^\top \boldsymbol{\xi}\right\}$$



Dantzig Wolfe Procedure

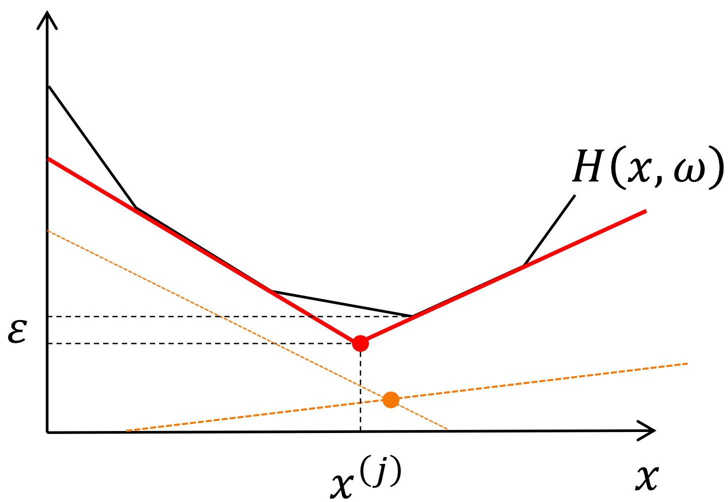
- The DWP enhances the CCG algorithm
- Both lower and upper bounds are tighter

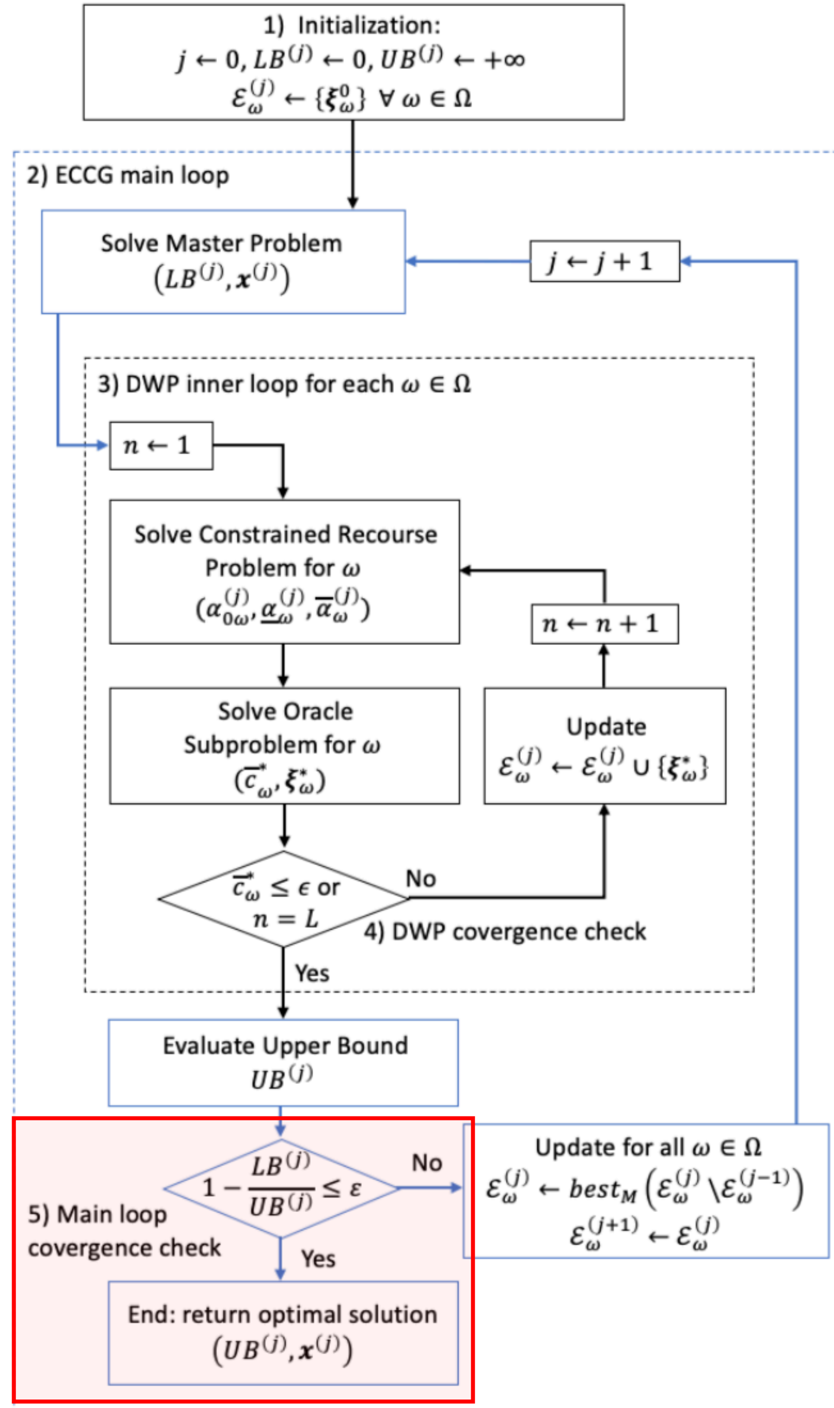




Methodological Contribution

- The DWP enhances the CCG algorithm
- Both lower and upper bounds are tighter
- ECCG: convergence is accelerated when
  - Master is heavy and Subproblem light
  - Select M and L parameters





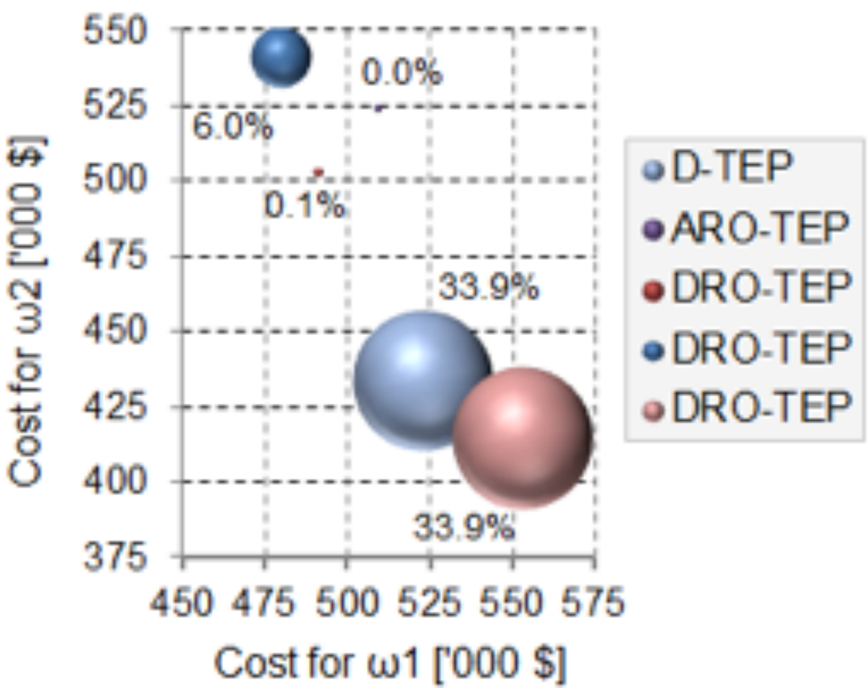
In this talk

- Motivation
- Multi-scale uncertainty
- Distributionally robust TEP model
- Enhanced column-and-constraint-generation algorithm
- Numerical experiments and discussion

Computational Experiments

TEP PROBLEMS SOLUTIONS AND OUT-OF-SAMPLE ASSESSMENT

Model	TEP solutions			Out-of-sample simulations											
				Low-variance distributions						High-variance distributions					
	New lines	Cost[10^4\$]		Normal distribution			Beta distribution			Normal distribution			Beta distribution		
		Inv.	Total	Cost[10^4\$] ω_1	Cost[10^4\$] ω_2	RI[%]	Cost[10^4\$] ω_1	Cost[10^4\$] ω_2	RI[%]	Cost[10^4\$] ω_1	Cost[10^4\$] ω_2	RI[%]	Cost[10^4\$] ω_1	Cost[10^4\$] ω_2	RI[%]
DRO(100,0)	6	11.3	60.3	48	52	2.5	47	50	0.0	50	55	10.6	48	54	6.0
DRO(0,100)	2	3.0	43.6	49	42	19.0	44	41	3.9	57	42	32.7	55	42	33.9
DRO(50,50)	8	12.2	58.0	49	50	0.4	48	50	0.0	51	50	4.0	49	50	0.1
ARO	8	14.3	160.0	51	52	0.3	50	52	0.0	52	53	3.5	51	52	0.0
DET	-	-	37.6	46	41	19.0	41	39	3.9	54	44	32.7	52	43	33.9

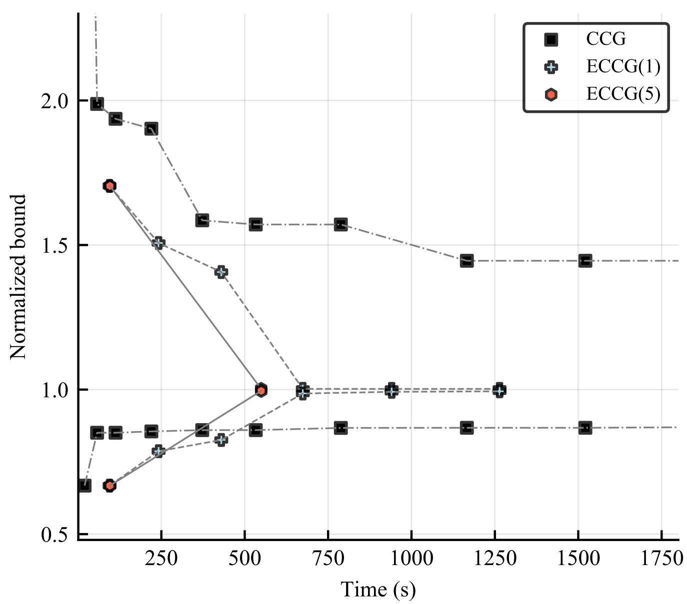


Computational Experiments

COMPARATIVE CPU TIMES (S) AND NUMBER OF ITERATIONS

Method	Uncertainty Vector Dimension							
	4		5		8		12	
	t. (s)	Iter.	t. (s)	Iter.	t. (s)	Iter.	t. (s)	Iter.
FVA	170	-	1,825	-	T	-	T	-
CCG	289	9	625	10	8,168	15	T	T
ECCG(1)	111	5	195	5	1,264	6	14,510	11
ECCG(2)	75	3	119	3	957	4	12,247	7
ECCG(3)	87	3	158	3	809	3	8,503	5
ECCG(4)	56	2	90	2	1,168	3	6,776	4
ECCG(5)	62	2	98	2	549	2	4,083	3

T - Time limit of 18 hours exceeded without convergence.



DETAILED ITERATION DATA FOR d=8

Method	Iter.	Time (s) Master	Inner.	Iter.	Accum.	GAP (%)	Inner iter. (#) ω_1	ω_2
CCG	1	13	7	20	20	85.1	1	1
	2	28	9	38	57	57.2	1	1
	3	47	9	55	113	56.1	1	1
	4	99	9	107	220	55.0	1	1
	5	142	11	153	373	45.8	1	1
	6	148	12	160	533	45.3	1	1
	7	244	11	255	788	44.8	1	1
	8	365	13	378	1,166	40.0	1	1
	9	343	12	355	1,521	40.0	1	1
	10	473	11	484	2,005	39.8	1	1
	11	760	13	773	2,778	35.4	1	1
	12	711	14	725	3,503	10.0	1	1
	13	1,319	18	1,337	4,839	7.4	1	1
	14	1,671	17	1,688	6,527	1.4	1	1
	15	1,623	18	1,641	8,168	0.7	1	1
ECCG(1)	1	13	81	95	95	60.8	16	20
	2	23	124	147	242	47.8	19	20
	3	60	128	188	430	41.3	19	20
	4	110	134	244	674	1.6	14	18
	5	147	119	266	940	1.0	13	14
	6	226	97	324	1,264	0.8	12	8
ECCG(2)	1	13	82	95	95	60.8	16	20
	2	48	142	190	286	33.6	20	20
	3	169	134	303	589	10.4	18	17
	4	289	78	368	957	0.6	5	12
ECCG(3)	1	14	83	96	96	60.8	16	20
	2	101	118	218	314	16.2	14	20
	3	386	109	495	809	0.7	10	14
ECCG(4)	1	14	83	97	97	60.8	16	20
	2	151	153	303	400	3.9	14	20
	3	685	82	768	1,168	0.2	9	7
ECCG(5)	1	14	82	96	96	60.8	16	20
	2	321	132	453	549	0.7	14	17

Computational Experiments

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Distributionally Robust Transmission Expansion Planning: a Multi-scale Uncertainty Approach

Alexandre Velloso, David Pozo, *Senior Member, IEEE*, Alexandre Street, *Senior Member, IEEE*

***Abstract*—We present a distributionally robust optimization (DRO) approach for the transmission expansion planning problem, considering both long- and short-term uncertainties on the system demand and renewable generation. On the long-term level, as it is customary in industry applications, the deep uncertainties arising from social and economic transformations, political and environmental issues, and technology disruptions are addressed by long-term scenarios devised by experts. The system planner is then allowed to consider exogenous long-term scenarios containing partial information about the random parameters, namely, the average and the support set. For each constructed long-term scenario, a conditional ambiguity set is used to model the incomplete knowledge about the probability distribution of the uncertain parameters in the short-term. Consequently, the mathematical problem is formulated as a DRO model with multiple conditional ambiguity sets. The resulting infinite-dimensional problem is recast as an exact, although very large, finite-deterministic mixed-integer linear programming problem. To circumvent scalability issues, we propose a new enhanced-column-and-constraint-generation (ECCG) decomposition approach with an additional Dantzig–Wolfe procedure. In comparison to existing methods, ECCG leads to a better representation of the recourse function and, consequently, tighter bounds. Numerical experiments based on the benchmark IEEE 118-bus system are reported to corroborate the effectiveness of the method.**

***Index Terms*—Ambiguity aversion, deep uncertainty, distributionally robust optimization, multi-scale uncertainty, renewable generation, stochastic optimization, transmission expansion planning.**

$\mathcal{P}_\omega$	Set of probability measures conditioned to the long-term scenario ω.
$\mathcal{S}, \mathcal{S}_\omega$	Sample space, and subset of the sample space associated with the long-term scenario ω.
$\mathbb{S}, \mathbb{S}_\omega$	Appropriated sigma-algebras for $\mathcal{S}$ and $\mathcal{S}_\omega$, respectively.
$\mathcal{T}$	Set of block of hours t.
$\mathcal{X}$	Set of feasible investment plans.

B. Functions

$\tilde{\xi}$	Measurable function (or random vector) modeling the uncertainty in the net demand.
$\tilde{\xi}_t$	Subvector of $\tilde{\xi}$ related to block of hour t.
$\tilde{\xi}(s)$	Realization of $\tilde{\xi}$ for scenario s.
$g(\mathbf{x}, \boldsymbol{\xi}, \omega)$	Minimum-cost dispatch function for investment $\mathbf{x}$, realization $\boldsymbol{\xi}$, and long-term scenario ω.
$H_{DR}(\mathbf{x}, \omega)$	Distributionally robust recourse function for investment $\mathbf{x}$, under the long-term scenario ω.
$\overline{H}_{DR}(\mathbf{x}, \omega)$	Upper bound function for $H_{DR}(\mathbf{x}, \omega)$.
$\underline{H}_{DR}(\mathbf{x}, \omega)$	Lower bound function for $H_{DR}(\mathbf{x}, \omega)$.

C. Constants and Parameters

ϵ, ε	Tolerance for the inner and the main loop (in monetary and percentage units, respectively).
$\lambda_\omega^{(-)}, \lambda_\omega^{(+)}$	Vectors of imbalance costs for the long-term scenario ω.

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