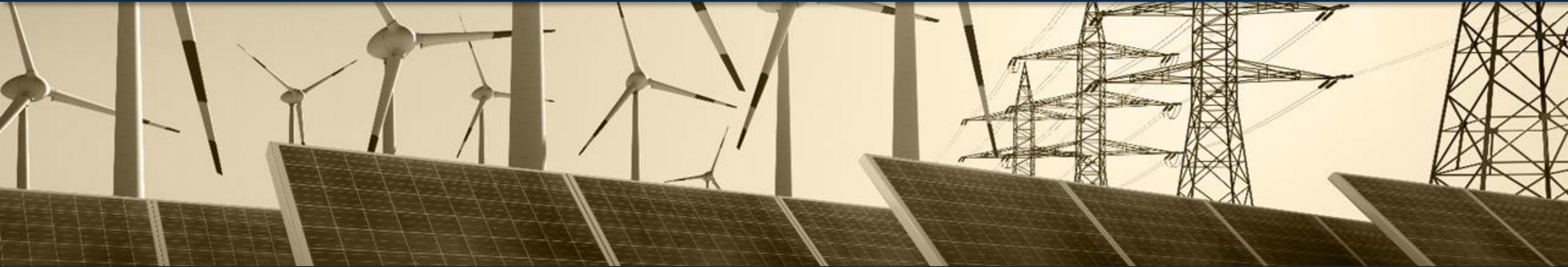
Hydrothermal scheduling using affine rules

Rodrigo Novaes

rnovaes@psr-inc.com

Joaquim Dias Garcia, Guilherme Machado, Gerson Couto, Mario Veiga Pereira, Bernardo Bezerra



PSR

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PSR

PSR integrates **consulting studies**, develops advanced **analytical tools** and research of new methodologies on **energy systems**

Our team has **85 people** with degrees in optimization, energy systems, statistics and computer/data science

We work in more than **70 countries** in all continents

*See our 2018 retrospective **here***

The context and the problem we want to solve

- Hydrothermal operation is a well-known large-scale and real-life application of Stochastic Programming

- Some challenges are:

  - Decision has to be made **before** knowing the real net load

  - The **variability** and the **non-linear** relationships of **renewable** sources

  - **Time coupling** constraints (ramp rates, min. uptime and downtime, outflows,...)

- The **conceptual** problem is

$$
\begin{aligned}
&\boldsymbol{Minimize}\ Total\ costs\\
&\boldsymbol{s.t.}\quad Hydro\ balance\\
&\qquad\ \ Load\ balance\\
&\qquad\ \ Operational\ constraints\\
&(unit\ commitment, generation\ and\ ramp\ constrains\ \ldots)
\end{aligned}
$$

The proposed framework

CLUSTERING THE SCENARIOS

- Calculate the deviations in net demand
- Clustering by severity
- Clustering for different regions

WEIGHTING THE SCENARIOS

- What scenarios are more meaningful?
- Modeling the decrease of the prediction power accross time

THE AFFINE RULES

- As simple as that: swap the actual variables by linear functions of the uncertain parameters

The affine rules

What is it?

- We'll start from the sample average approximation stochastic programming problem

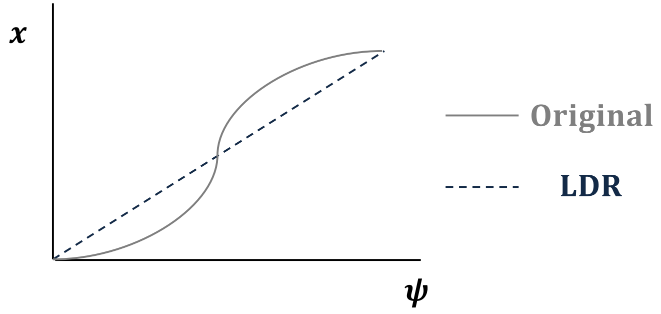
$$Minimize \frac{1}{|\Psi|} \sum_{\psi \in \Psi} \sum_{t=1}^{T} c_t' \, x_t$$

$$s.t. \quad Ax_t(\psi_t) \leq b(\psi_t) \qquad \forall \, \psi_t \in \Psi$$

- The whole idea is to transform that problem into this one

$$Minimize \frac{1}{|\Psi|} \sum_{\psi \in \Psi} \sum_{t=1}^{T} c_t' \, (\alpha_t \psi_t + \beta_t)$$

$$s.t. \quad A(\alpha_t \psi_t + \beta_t) \leq b(\psi_t) \qquad \forall \, \psi_t \in \Psi$$



The affine rules
How to use them in hydrothermal scheduling?

- Somehow you must get a group of **net load** scenarios
- Choose a **reference** scenario and solve the **original** problem
- Using the decisions made to this reference scenario, construct the affine rules to the other scenarios
- For example, suppose a load balance equation for the scenario $\boldsymbol{s_0}$:

$$\boldsymbol{g}^{\mathbf{0}} + \boldsymbol{\rho} \times \boldsymbol{u}^{\mathbf{0}} = \boldsymbol{d}^{\mathbf{0}}$$

- For a given $\boldsymbol{s}$ scenario you could write:

$$\boldsymbol{\alpha_1}\left(\boldsymbol{d}^{s} - \boldsymbol{d}^{\mathbf{0}}\right) + \boldsymbol{g}^{\mathbf{0}} + \boldsymbol{\rho} \times \left[\boldsymbol{\alpha_2}\left(\boldsymbol{d}^{s} - \boldsymbol{d}^{\mathbf{0}}\right) + \boldsymbol{u}^{\mathbf{0}}\right] = \boldsymbol{d}^{s}$$

The affine rules

Why did we do that?

- **Reduce** the number of variables and **complexity** (i.e., computational time)
- That's a **way to solve** a multistage stochastic **hydrothermal scheduling** problem with **unit commitment**
- If the net load **scenarios change**, under some conditions, you can still **use the same set** of affine rules already calculated to obtain the generation levels
- It's **like** if you could calculate the **basis** of this problem, as you can in linear programming:

$$\boldsymbol{x_b} = \boldsymbol{B}^{-1} \times \boldsymbol{b}$$

Clustering the scenarios

- **Why is it necessary?**

Because the **variability** of the renewables can make the problem **infeasible**

- **How did we do that?**

Step 1: Calculate the deviations in **net demand** for each time step and scenario $\boldsymbol{s}$, comparing to $\boldsymbol{s_0}$

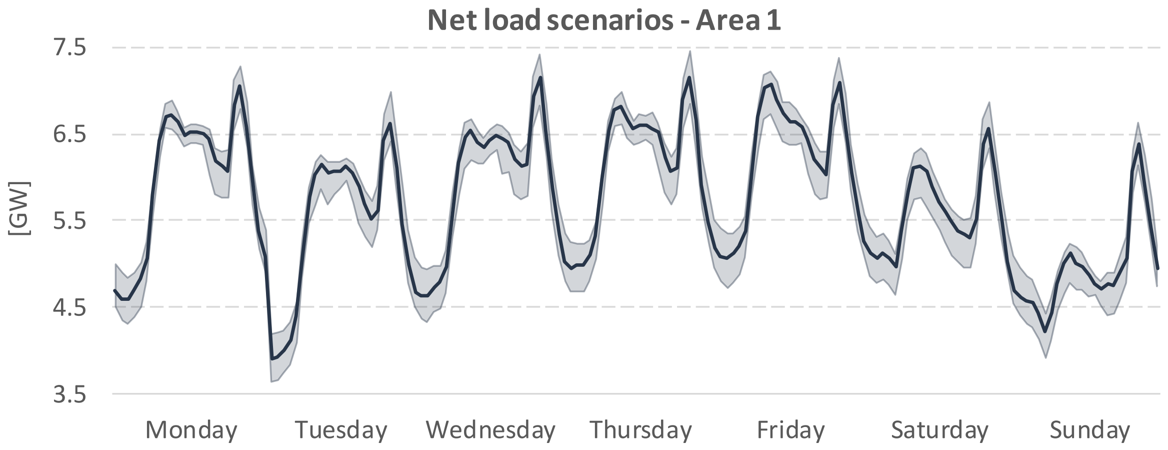
Step 2: Using theses deviations, calculate a scalar, for each scenario

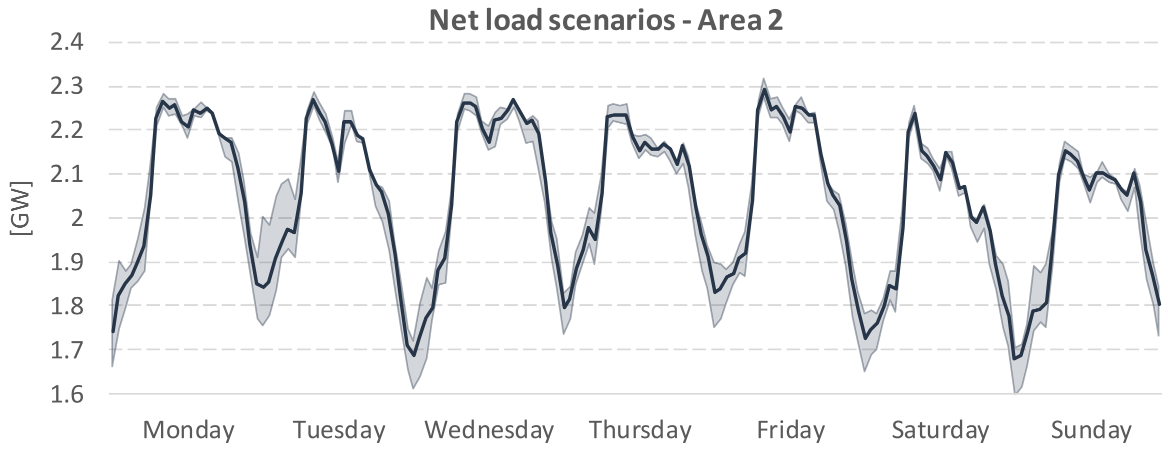
Step 3: Group the scalars (use *k-means* if there is more than one area)

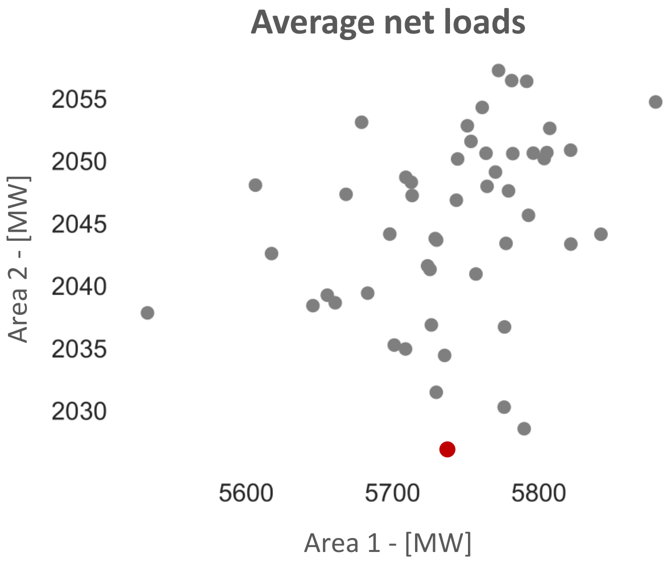
Step 4: Solve the optimization problem for each of the clusters

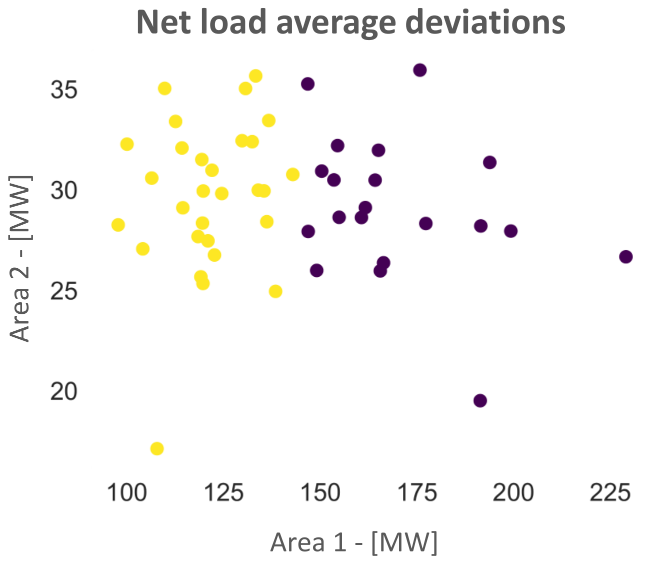
Clustering the scenarios

50 scenarios from the Chilean system: 2 areas and 2 clusters









Weighting the scenarios

- **Why is it necessary?**

Because your scenarios set could be not so large as we are used to deal with in the long term planning

- **How did we do that?**

Calculating a Gaussian distribution for a **given scenarios set** and applying it to each scenario at each hour where μ_0 is the mean and ω_0 is the standard deviation of the reference scenario. d_h^i is the **net load** for the **system** and σ_h^i is the weight for scenario *i* and hour *h*.

$$\boldsymbol{f(x) = \frac{1}{\omega_0\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\omega_0}\right)^2}} \qquad \boldsymbol{\sigma_h^i = \frac{f(d_h^i)}{\sum_s f(d_h^s)}}$$

- You may also model that the **accuracy** of net load **forecasts decreases** as we increase the time horizon

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www.psr-inc.com
psr@psr-inc.com
+55 21 3906-2100

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Thank you!

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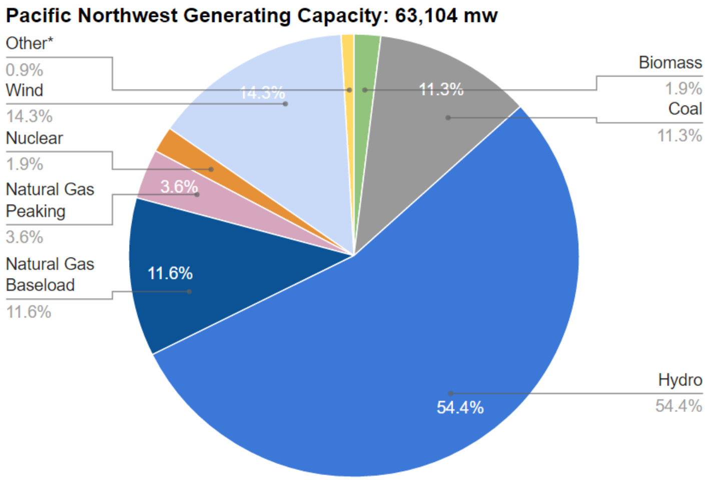
Appendix

Some information about the Genesys project

The Pacific Northwest system



- **Plants to be represented**

76 hydros (38 reservoirs), 133 thermals

339 non-dipatchable plants (including renewables)

- **Network**

DC optimal power flow, 34 nodes, 74 circuits

Big challenges

- **54 milllion MIPs problems**

 Daily/weekly operation with hourly resolution

- **6000 scenarios (8760 hours each)**

- **Distributed processing**

 500 servers

- **Distributed storage**

 10 Tbytes for results and statistics

- **In 8 hours!**

Some information about the Genesys project