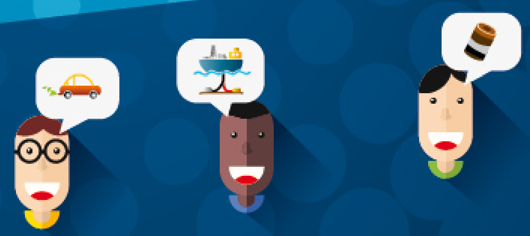
Modelos Matemáticos de Otimização Sob Incertezas

Project OTIM-PBR: A Stochastic Programming Approach to Brazilian Oil Supply Chain Planning

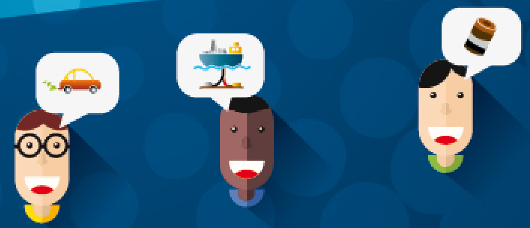
Agenda

1. Project Team
2. Brazilian Oil Supply Chain
3. Current Approach
4. Uncertain Sources
5. Proposed Approach
    a. Statistical Model
    b. Optimization Model
6. Conclusions

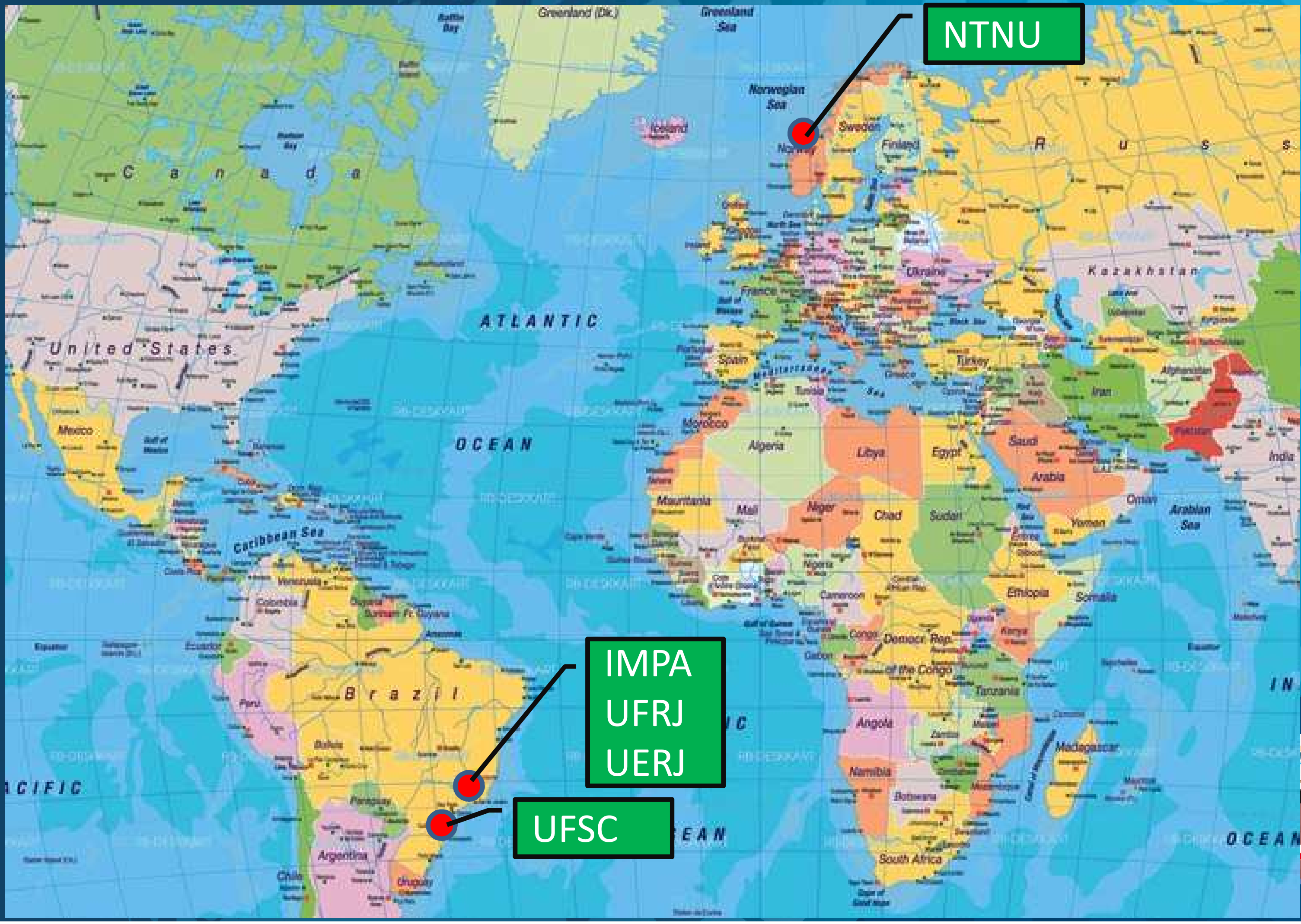


Agenda

1. Project Team
2. Brazilian Oil Supply Chain
3. Current Approach
4. Uncertain Sources
5. Proposed Approach
    a. Statistical Model
    b. Optimization Model
6. Conclusions

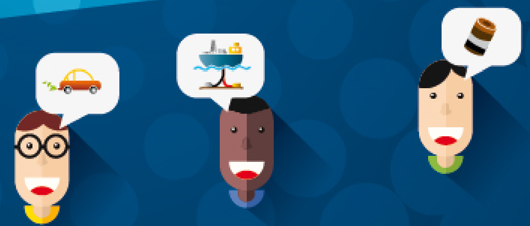


OTM-PBR: Academic Team

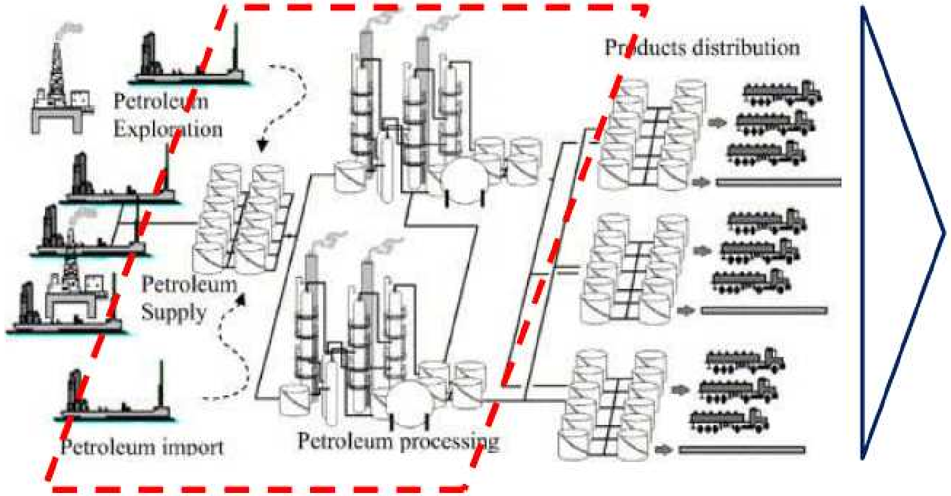


Agenda

1. Project Team
2. Brazilian Oil Supply Chain
3. Current Approach
4. Uncertain Sources
5. Proposed Approach
   a. Statistical Model
   b. Optimization Model
6. Conclusions



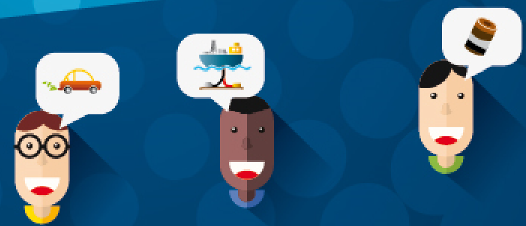
The setting



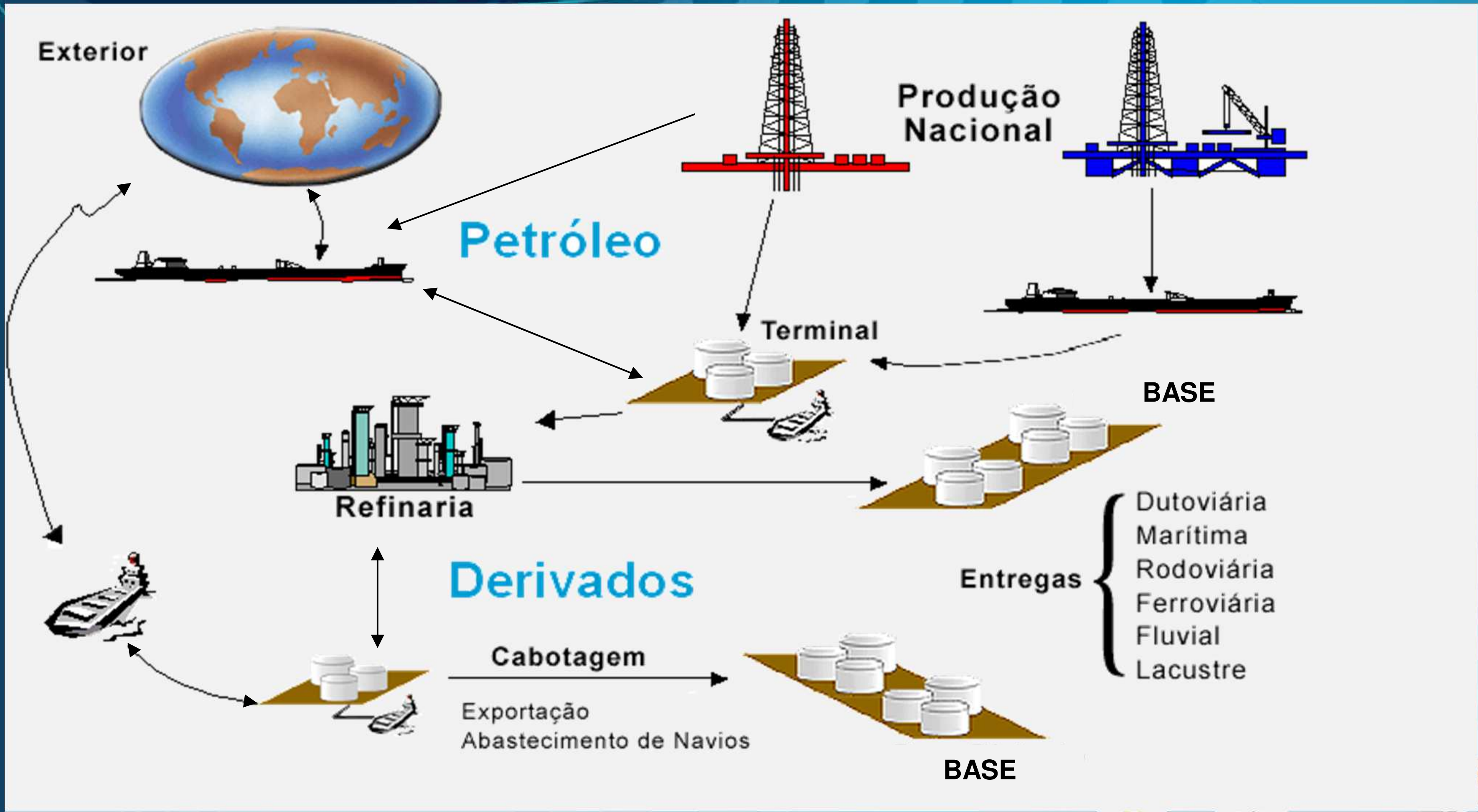
Wide variety of comercial, industrial and logistics operations occur over the midstream segment

Strong dependence among the operations and some gains can only be estimated by considering the whole supply chain

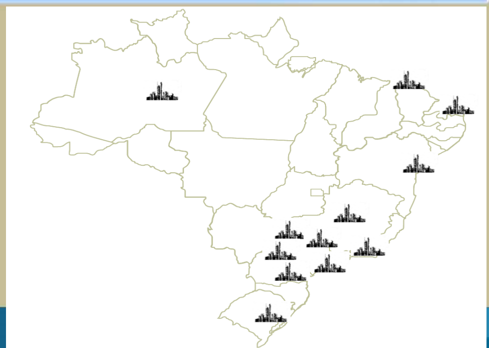
Planning this segment is crucial to achieve the success of all operations.



Brazil's supply chain



Brazil's supply chain



Curde Oil:

- Imports (1)
- Exports (3)
- Production (5)

Oil Products:

- Imports (2)
- Exports (4)
- Market Selling (6)

~ 200 different crude oils

~ 50 different oil products

10 basins of oil production

23000 km pipelines

>100 platforms

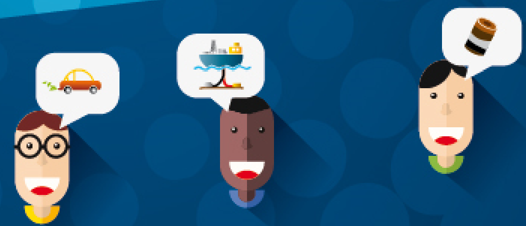
> 40 terminals in Brazil

>200 terminals abroad

13 refineries

Agenda

1. Project Team
2. Brazilian Oil Supply Chain
3. Current Approach
4. Uncertain Sources
5. Proposed Approach
    a. Statistical Model
    b. Optimization Model
6. Conclusions



Supply chain planning tool Computational model

PlanAb

- Computational model for Petrobras supply chain
- Tool to determine the optimal planning for the whole supply chain of the company for the next month.

- Currently PlanAb is a deterministic model that solves a large scale linear programming problem

$$
\begin{cases}
\min_{x,y} & c^\top x + q^\top y & \\
\text{s.t} & Ax & = b \\
& Tx + Wy & = h \\
& \underline{x} \le x \le \overline{x} & \\
& \underline{y} \le y \le \overline{y} & \\
& y_i = \bar{y}_i & i \in I_V
\end{cases}
$$

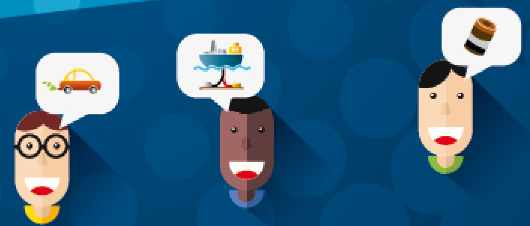
Supply chain planning tool Computational model

PlanAb

$$
\begin{cases}
\min_{x,y} & c^\top x + q^\top y \\
\text{s.t} & Ax = b \\
& Tx + Wy = h \\
& \underline{x} \leq x \leq \overline{x} \\
& \underline{y} \leq y \leq \overline{y} \\
& \boxed{y_i = \bar{y}_i \quad i \in I_V}
\end{cases}
$$

Availability of oil volume from the plataforms

- Variable x represents commercial transactions such as imports along the planning horizon (four months)
- Variable y represents exports and logistic operations such transportation and refinery oil/derivative production



Most relevant outputs

Commercial transactions

Oil

- Imports
- Exports

Derivatives

- Imports
- Exports

Logistic operations

Refineries

- Type of consumed oil
- Type of produced derivatives

National market

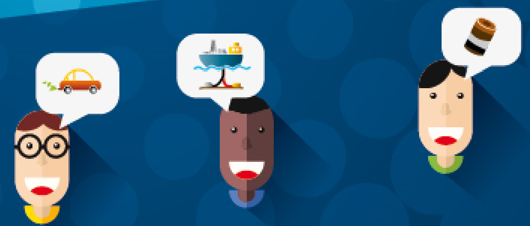
- Which refinery sends products to which market

Objective function
PlanAb model

Current model: $\min_{x,y} \; c^\top x + q^\top y$

Model (objective function)

Costs	Revenues
•Oil imports	•Oil exports
•Derivative imports	•Derivative exports
•Derivative storage	•Derivative national sales
•Refining unit operations	
•Transportation	

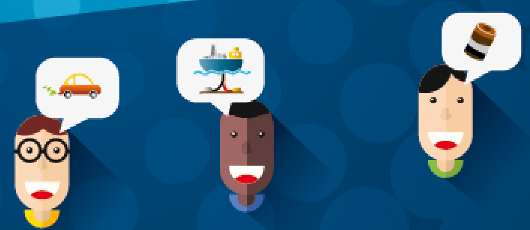


Variables and constraints
PlanAb model

1. Oil and derivatives balance
   - Includes all constraints involving the system balance of oil/derivatives and how they flow in the network
   - Balancing the derivatives in each terminal and refinery

$$
\begin{aligned}
Ax & & &= b \\
Tx &+ Wy &&= h
\end{aligned}
$$

$$
\begin{aligned}
\underline{y} \leq y &\leq \overline{y} \\
y_i = \bar{y}_i & \quad i \in I_V
\end{aligned}
$$

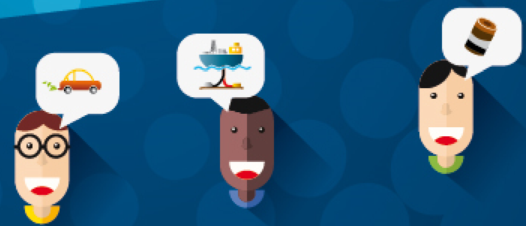


PlanAb

- Deterministic

$$\left\{\begin{array}{llllll} \min_{x,y} & c^\top x & + & q^\top y & & \\ \text{s.t} & Ax & & & = & b \\ & Tx & + & Wy & = & h \\ & \underline{x} \leq & x & \leq \overline{x} & & \\ & \underline{y} \leq & y & \leq \overline{y} & & \\ & y_i & = & \bar{y}_i & & i \in I_V \end{array}\right.$$

How large are these models?



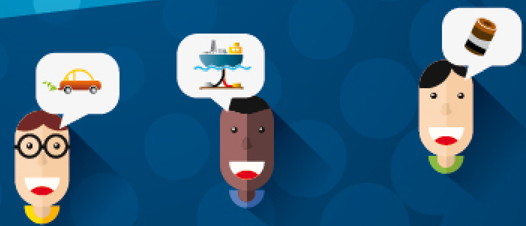
Deterministic PlanAb

Implemented in AIMMS

- LP's dimension:
  - 2.3 million of variables
  - 1.7 million of constraints

- Constraints' matrix:
  - Sparse (0,0002% non-zero elements; approximately 7.3 million of elements)
  - Block-diagonal structure

$$\begin{cases} \min_{x,y} & c^\top x & + & q^\top y & & \\ \text{s.t} & Ax & & & = b & \\ & Tx & + & Wy & = h & \\ & \underline{x} \leq & x & \leq \overline{x} & & \\ & \underline{y} \leq & y & \leq \overline{y} & & \\ & y_i & = & \bar{y}_i & & i \in I_V \end{cases}$$



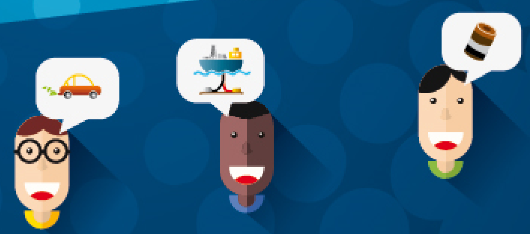
Deterministic PlanAb

- PlanAb LP:
  - 2.3 million of variables, 1.7 million of constraints
  - 7.3 million of non-zero elements (0.0002% - sparse matrix)
  - After presolve, problem size reduced approximately by 80%

Model	Variables	Constraints	Non-zero elements	Presolve time (sec)
Original	2.3 million	1.7 million	7.3 million	0
Conservative presolve	644,000	307,000	2.88 million	2.7
Automatic presolve	385,000	94,000	2.19 million	11.1
Aggressive presolve	385,000	92,000	2.14 million	13.2

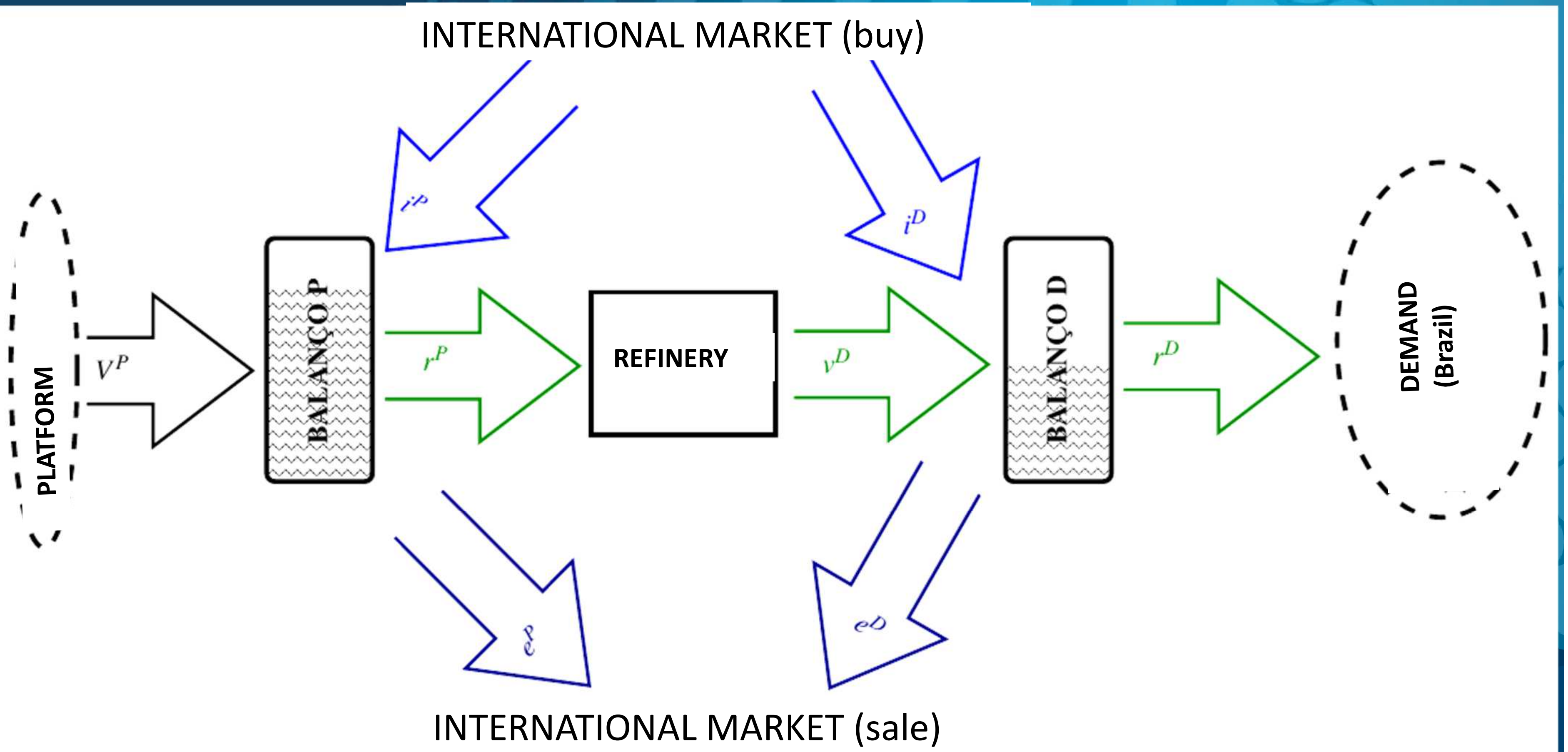
Agenda

1. Project Team
2. Brazilian Oil Supply Chain
3. Current Approach
4. Uncertain Sources
5. Proposed Approach
   a. Statistical Model
   b. Optimization Model
6. Conclusions



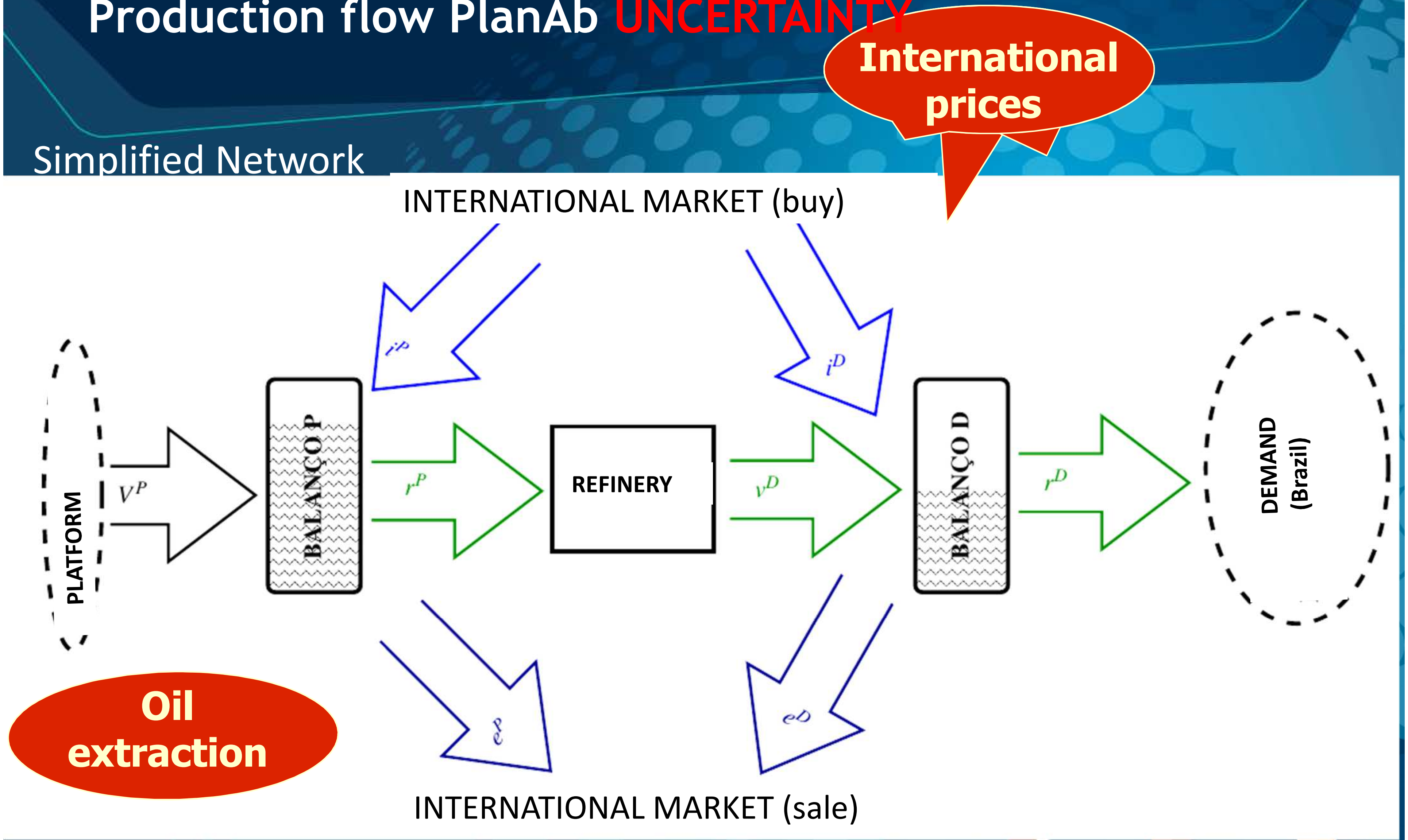
Production flow PlanAb

Simplified Network



Production flow PlanAb UNCERTAINTY

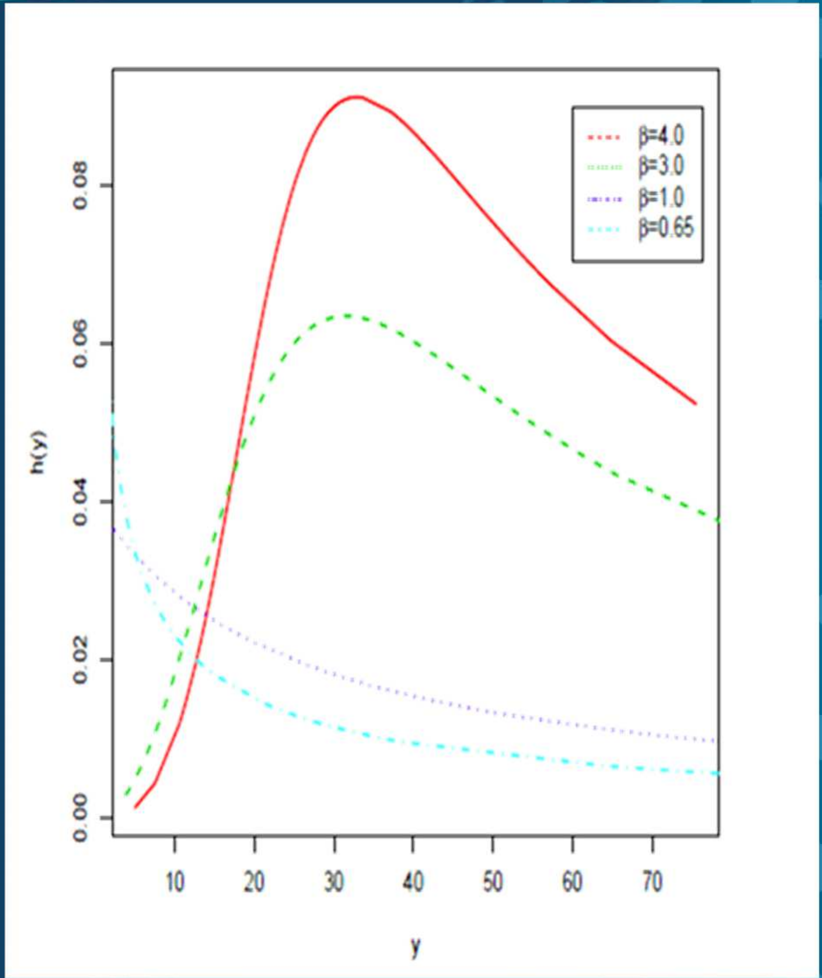
Simplified Network



Oil Production

- Naturally oil fields have a production curve along the time.
  - Some technics are applied to smoothing the curve, the result of this technics are not deterministic
- Maintenances whose duration is subject to uncertainties

Oil Production
Log-logistic Distribution



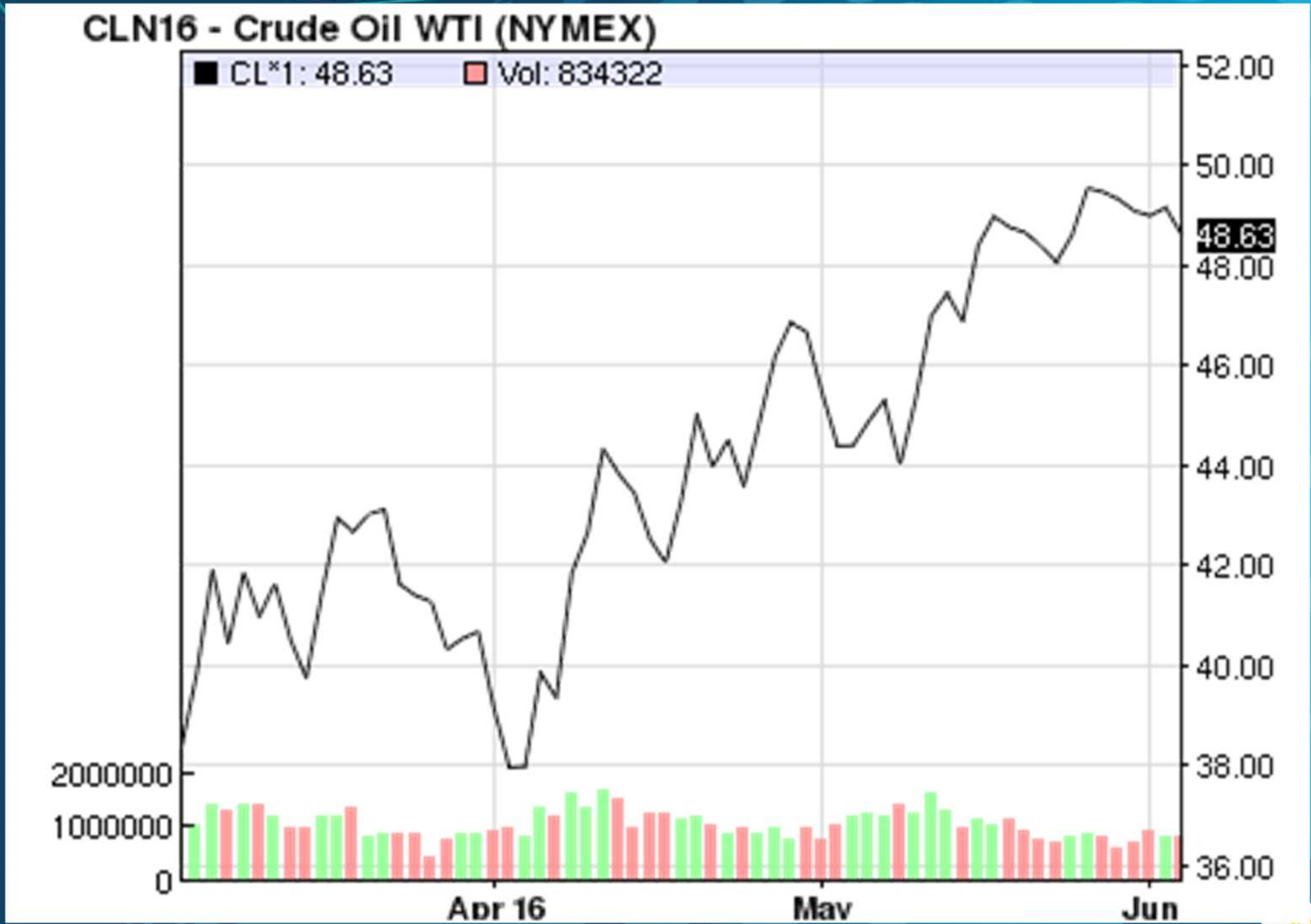
International Prices

- Volatility is a natural attribute of the historical curve of oil and derivatives prices, with high uncertainty.

International Prices

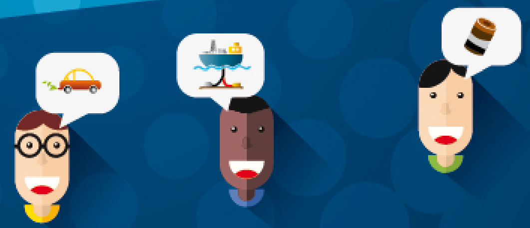
International Prices

International Prices



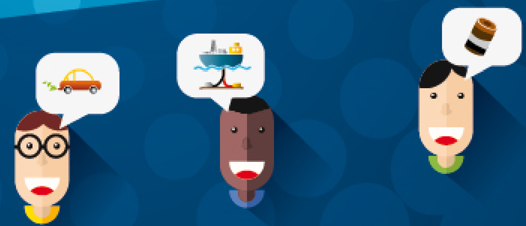
Uncertainly - Impact

	Atmospheric Distillation 1						Atmospheric Distillation 2					
	QAV		Naphtha		Diesel		QAV		Naphtha		Diesel	
	% Vol	% S	% Vol	% S	% Vol	% S	% Vol	% S	% Vol	% S	% Vol	% S
Oil 1	9,6	0,178	16,4	0,028	28,0	1,516	13,3	0,145	5,0	0,020	30,0	1,071
Oil 2	6,2	0,203	3,6	0,015	26,9	0,586	7,6	0,179	1,1	0,003	29,0	0,499
Oil 3	3,2	0,228	2,3	0,035	22,8	0,514	4,2	0,207	0,2	0,011	24,8	0,452
Oil 4	5,4	0,199	5,7	0,011	25,9	0,591	7,0	0,174	1,4	0,000	27,9	0,504
Oil 5	5,7	0,259	5,4	0,014	27,7	0,596	7,2	0,224	1,5	0,003	29,7	0,517
Oil 6	11,2	0,015	26,2	0,009	24,0	0,050	15,6	0,016	9,0	0,006	25,4	0,037



Agenda

1. Project Team
2. Brazilian Oil Supply Chain
3. Current Approach
4. Uncertain Sources
5. Proposed Approach
   a. Statistical Model
   b. Optimization Model
6. Conclusions



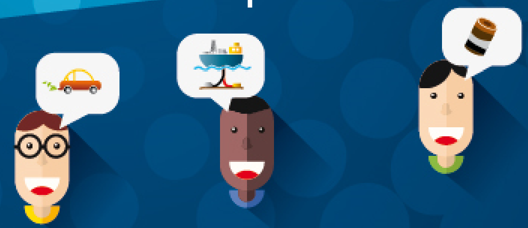
Stochastic Optimization

– Two- Stage Stochastic Linear Program with Recourse

- Divides the decision variables in two stages.
- First stage variables must be decided prior to uncertainties.
- Second stage variables correct the solution after the completion of uncertainties.

$$\underset{x \in X}{Min} \left\{ z(x) = c^T x + \sum_{s=1}^{S} p_s Q_s(x, \xi_s) \right\}$$

- In the first stage, the decision is *here and now*, prior to the uncertainties.
- The decision taken at the second stage problem reflects the optimal behavior at the time uncertainty is disclosed



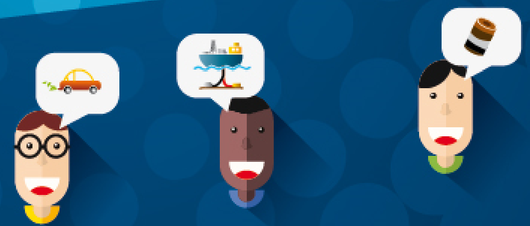
Stochastic Optimization
Measuring an optimization under uncertainties

- Expected (or mean) value problem: $EV = \min_x z(x, \bar{\xi}), \bar{\xi} = \mathbb{E}[\xi]$

- The expected result of using the EV solution measures the performance of $\bar{x}(\bar{\xi})$ (optimal second-stage reactions given $\bar{x}(\bar{\xi})$):

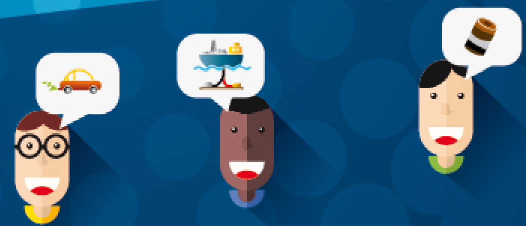
$$EEV = \mathbb{E}_{\xi}[z(\bar{x}(\bar{\xi}), \xi)]$$

- Value of the Stochastic Solution – VSS
  - $\boldsymbol{VSS = EEV - RP}$



Computational model

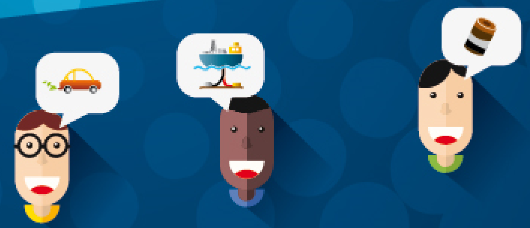
- Block 1: Statistical
  - Tool generating scenarios for:
  - International prices (oil and derivatives, correlated)
  - Oil volume arriving from the platforms each month.
  - Technique: multivariate model + Kalman filter + SDP NLP solver, assessed by backtesting .
- Block 2: Optimization
  - Tool to determine the optimal policy for the whole supply chain of the company for the next month. It includes:
  - To which extent the company network (of production, transportation, commercialization) can be simplified?
  - How to handle uncertainty in the optimization problem (in the cost and in the right hand side Problem solution via decomposition method huge scale problem)



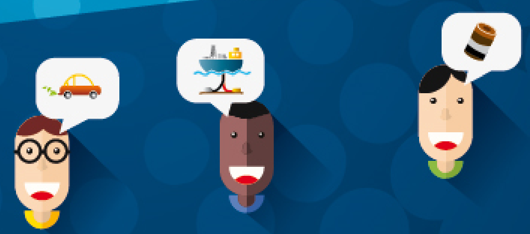
Some comments

- Uncertainty increases the (already huge) size of the linear program (LP).
- Mounting the data for the deterministic model takes longer than solving the LP.
- Solving an aggregate model is not an option: keeping the level of detail of the deterministic model is important for the company.

- BUT
  - Changing CPLEX stopping tolerance from default 10^{-8} to 10^{-4} provides a good trade-off between accuracy and solving time:
    - mean relative error on variables 0,007%
    - solving times reduced in 18,56%



Statistical Block



Sources of uncertainty
Calibration of price models

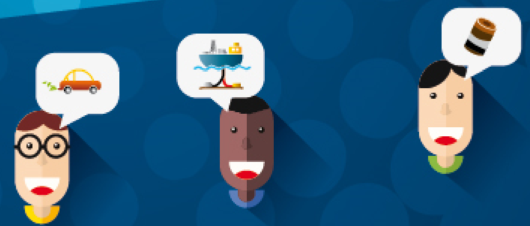
1. Oil, gasoline and diesel international prices

- The model is a process defined by stochastic differential equations.
- To determine the parameters defining the model (mean, trend, volatility, correlations) we maximize the proximity to historical data (likelihood) using a Kalman filter.
- Small scale non-convex optimization problem, solved by a nonlinear programming method of interior feasible directions.

Sources of uncertainty
Calibration of price models

- They should include important oil price features such as picks, seasonality, mean reversion, etc.
- There are several stochastic processes for modelling commodity prices.
- The models are completely described by certain parameters that can be time dependent (mean, trend, volatility, correlations) and whose values need to be estimated (calibration).

Optimization Block



Supply chain planning tool Computational model

PlanAb

- Computational model for Petrobras supply chain
- Tool to determine the optimal planning for the whole supply chain of the company for the next month.

- Currently PlanAb is a deterministic model that solves a large scale linear programming problem

$$
\left\{
\begin{array}{lllllll}
\min_{x,y} & c^\top x & + & q^\top y & & \\
\text{s.t} & Ax & & & = b & \\
 & Tx & + & Wy & = h & \\
 & \underline{x} \leq & x & \leq \overline{x} & & \\
 & \underline{y} \leq & y & \leq \overline{y} & & \\
 & y_i & = & \bar{y}_i & & i \in I_V
\end{array}
\right.
$$

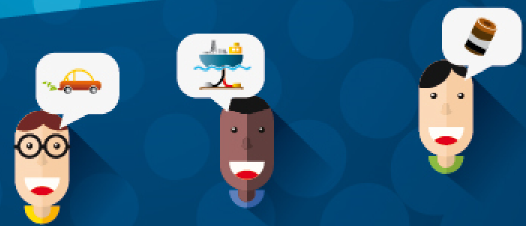
Supply chain planning tool Computational model

PlanAb

$$\begin{cases} \min_{x,y} & c^\top x + q^\top y \\ \text{s.t} & Ax = b \\ & Tx + Wy = h \\ & \underline{x} \le x \le \overline{x} \\ & \underline{y} \le y \le \overline{y} \\ & \boxed{y_i = \bar{y}_i \quad i \in I_V} \end{cases}$$

Availability of oil volume from the plataforms

- Variable x represents commercial transactions such as imports along the planning horizon (four months)
- Variable y represents exports and logistic operations such transportation and refinery oil/derivative production



Objective function
PlanAb model

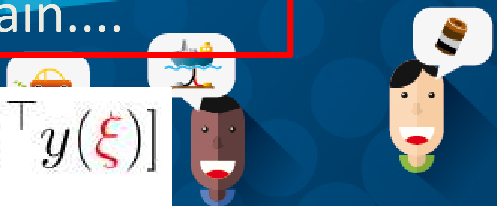
Current model: $$\min_{x,y} \; c^\top x \; + \; q^\top y$$

Model (objective function)

Costs	Revenues
•Oil imports •Derivative imports •Derivative storage •Refining unit operations •Transportation	•Oil exports •Derivative exports •Derivative national sales

Prices are uncertain....

Proposed model: $$\min_{x,y(\xi)} \; c^\top x + \mathbb{E}[q(\xi)^\top y(\xi)]$$



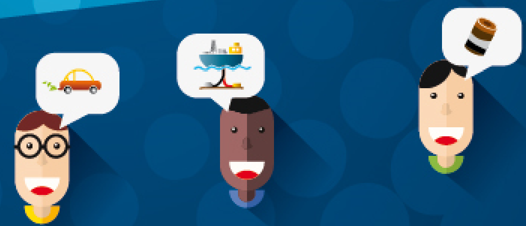
Variables and constraints
PlanAb model

1. Oil and derivatives balance
   - Includes all constraints involving the system balance of oil/derivatives and how they flow in the network
   - Balancing the derivatives in each terminal and refinery

$$\begin{aligned} Ax & & = b \\ Tx & + Wy & = h \end{aligned}$$

   - Considering adjustments on the available oil for:
     - Processing in the refineries
     - Exportation, when the volume of oil arriving from the platforms is larger than foreseen

$$\begin{aligned} \underline{y} \leq\ & y \leq \overline{y} & \\ y_i\ & = \bar{y}_i & i \in I_V \end{aligned}$$



Sources of uncertainty

1. Oil, gasoline and diesel international prices
2. Stochastic process: multivariate Schwartz-Smith

This uncertainty is represented by

$$\xi$$

1. Availability of national oil
2. The ratio between the foreseen and observed volumes follows a log-logistic probability distribution

This uncertainty is represented by ω

Two different sources of uncertainty!

PlanAb

- Deterministic

$$\begin{cases} \min_{x,y} & c^\top x & + & q^\top y & \\ \text{s.t} & Ax & & & = b \\ & Tx & + & Wy & = h \\ & \underline{x} \leq & x & \leq \overline{x} & \\ & \underline{y} \leq & y & \leq \overline{y} & \\ & y_i & = & \bar{y}_i & i \in I_V \end{cases}$$

How large are these models?

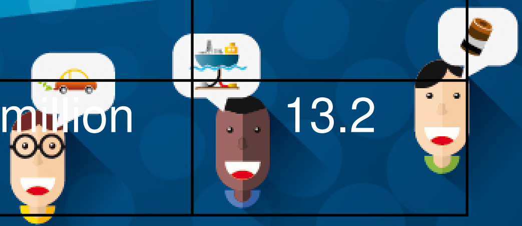
- Stochastic

$$\begin{cases} \min\limits_{x,y} & c^\top x & + & \mathbb{E}[q(\xi)^\top y(\xi,\omega)] & \\ \text{s.t} & Ax & & & = b \\ & Tx & + & Wy(\xi,\omega) & = h \\ & \underline{x} \leq & x & \leq \overline{x} & \\ & \underline{y} \leq & y(\xi,\omega) & \leq \overline{y} & \\ & y_i(\xi,\omega) & = & \bar{y}_i(\omega) & i \in I_V \\ & & & & \\ & \forall\, \xi \in \Xi & & \omega \in \Omega & \end{cases}$$

Deterministic PlanAb

- PlanAb LP:
  - 2.3 million of variables, 1.7 million of constraints
  - 7.3 million of non-zero elements (0.0002% - sparse matrix)
  - After presolve, problem size reduced approximately by 80%

Model	Variables	Constraints	Non-zero elements	Presolve time (sec)
Original	2.3 million	1.7 million	7.3 million	0
Conservative presolve	644,000	307,000	2.88 million	2.7
Automatic presolve	385,000	94,000	2.19 million	11.1
Aggressive presolve	385,000	92,000	2.14 million	13.2



Stochastic PlanAb

$$
\begin{cases}
\min_{x,y} & c^\top x & + & \mathbb{E}[q(\xi)^\top y(\xi,\omega)] & \\
\text{s.t} & Ax & & & = b \\
& Tx & + & Wy(\xi,\omega) & = h \\
& \underline{x} \leq & x & \leq \overline{x} & \\
& \underline{y} \leq & y(\xi,\omega) & \leq \overline{y} & \\
& y_i(\xi,\omega) & = & \bar{y}_i(\omega) & i \in I_V \\
& & & & \\
& \forall\, \xi \in \Xi & & \omega \in \Omega &
\end{cases}
$$

- Problem's size depends on the number of scenarios of price and oil volume
  - Number of variables is N times 2.3 million
  - Number of constraints is N times 1.7 million

- Impossible to load the problem in a (powerful) computer for N>10! (Memory issues) $N = |\Xi| * |\Omega|$

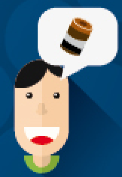
Two-stage decomposition
Stochastic PlanAb

- Consider finitely many scenarios of price and oil volume
- The problem is decomposed into two decision levels

with

$$\begin{cases} \min\limits_{x} & c^\top x \; + \; \mathbb{E}[Q(x;\xi,\omega)] \\ \text{s.t} & Ax \; = b \\ & \underline{x} \leq \; x \; \leq \overline{x} \end{cases}$$

$$Q(x;\xi,\omega) \begin{cases} \min\limits_{y} & q(\xi)^\top y \\ \text{s.t} & Wy \; = h - Tx \\ & \underline{y} \leq \; y \; \leq \overline{y} \\ & y_i \; = \bar{y}_i(\omega) \quad i \in I_V \end{cases}$$



Two-stage decomposition
Stochastic PlanAb

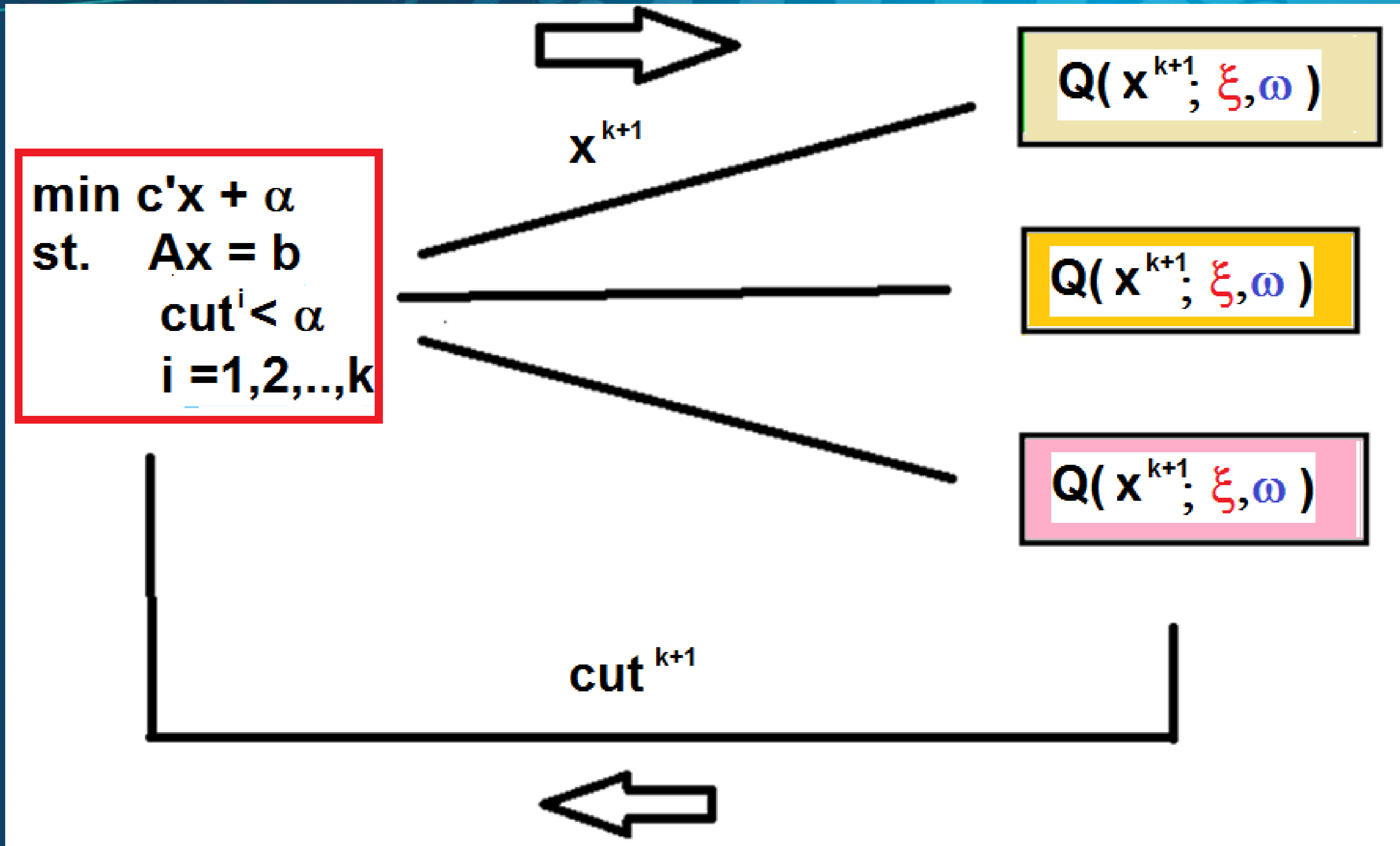
- Consider finitely many scenarios of price and oil volume
- The problem is decomposed into two decision levels

with

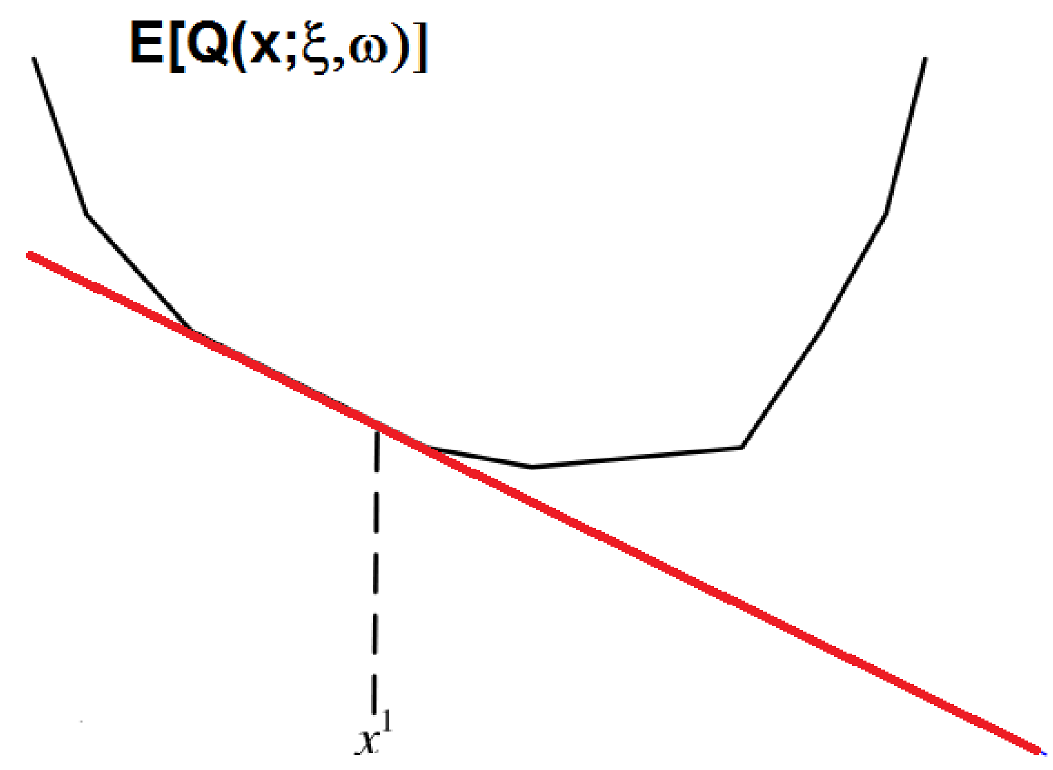
$$\begin{cases} \min\limits_{x} & c^\top x \; + \; \sum_{i=1}^{N} p_i [Q(x; \xi_i, \omega_i)] \\ \text{s.t} & Ax \; = \; b \\ & \underline{x} \leq \; x \; \leq \overline{x} \end{cases}$$

$$Q(x; \xi, \omega) \begin{cases} \min\limits_{y} & q(\xi)^\top y \\ \text{s.t} & Wy \quad = \; h - Tx \\ & \underline{y} \leq \quad y \; \leq \overline{y} \\ & y_i \quad\quad = \; \bar{y}_i(\omega) \qquad i \in I_V \end{cases}$$

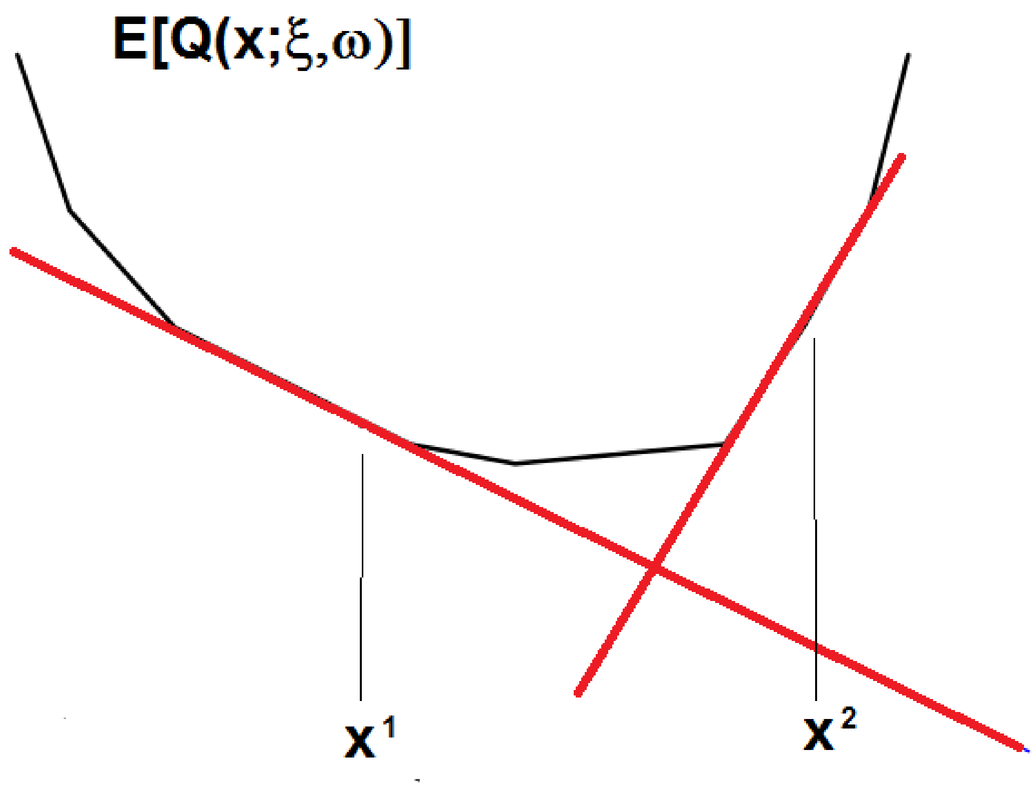
Two-stage decomposition
Stochastic PlanAb



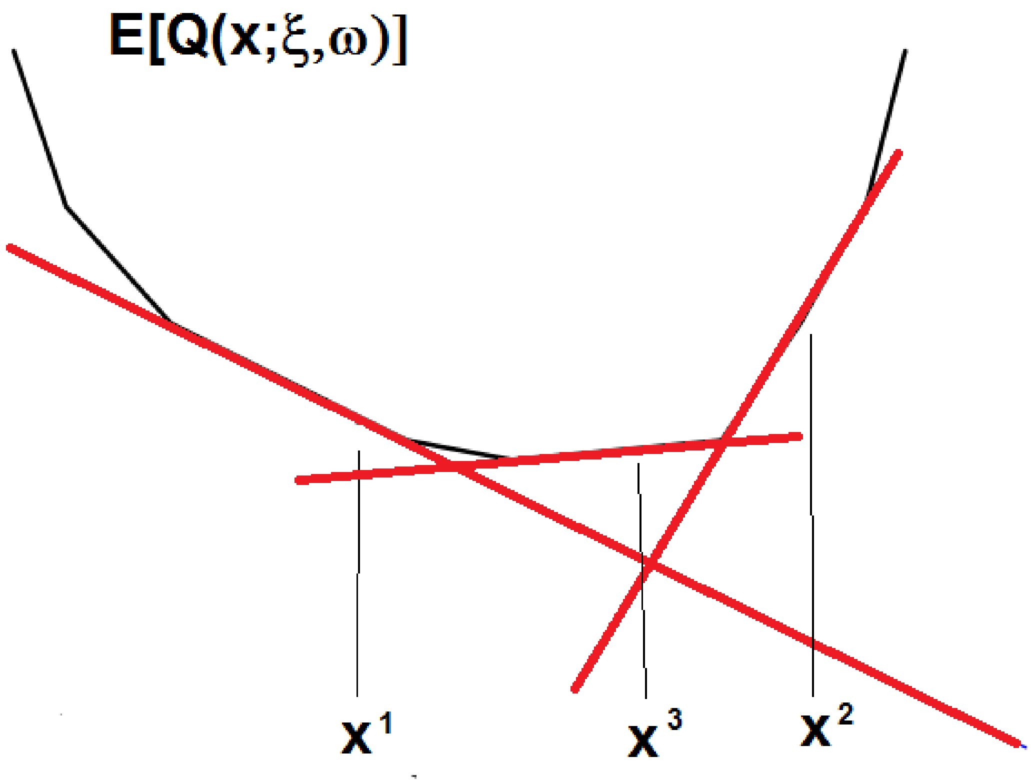
Cutting-plane approximation



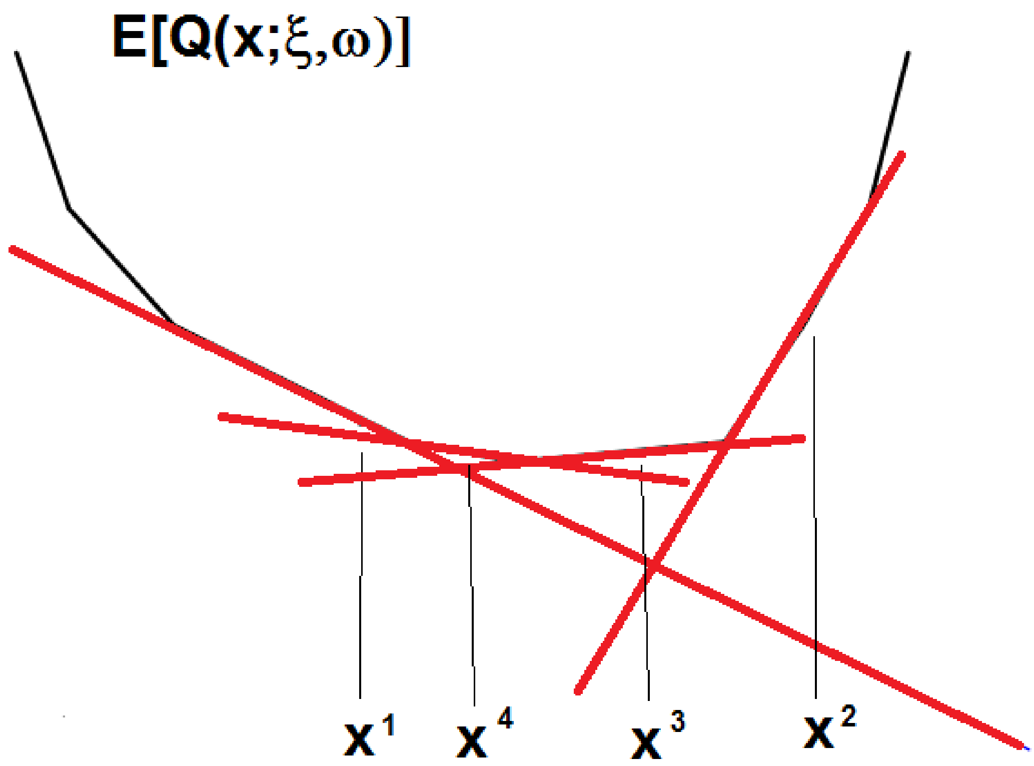
Cutting-plane approximation



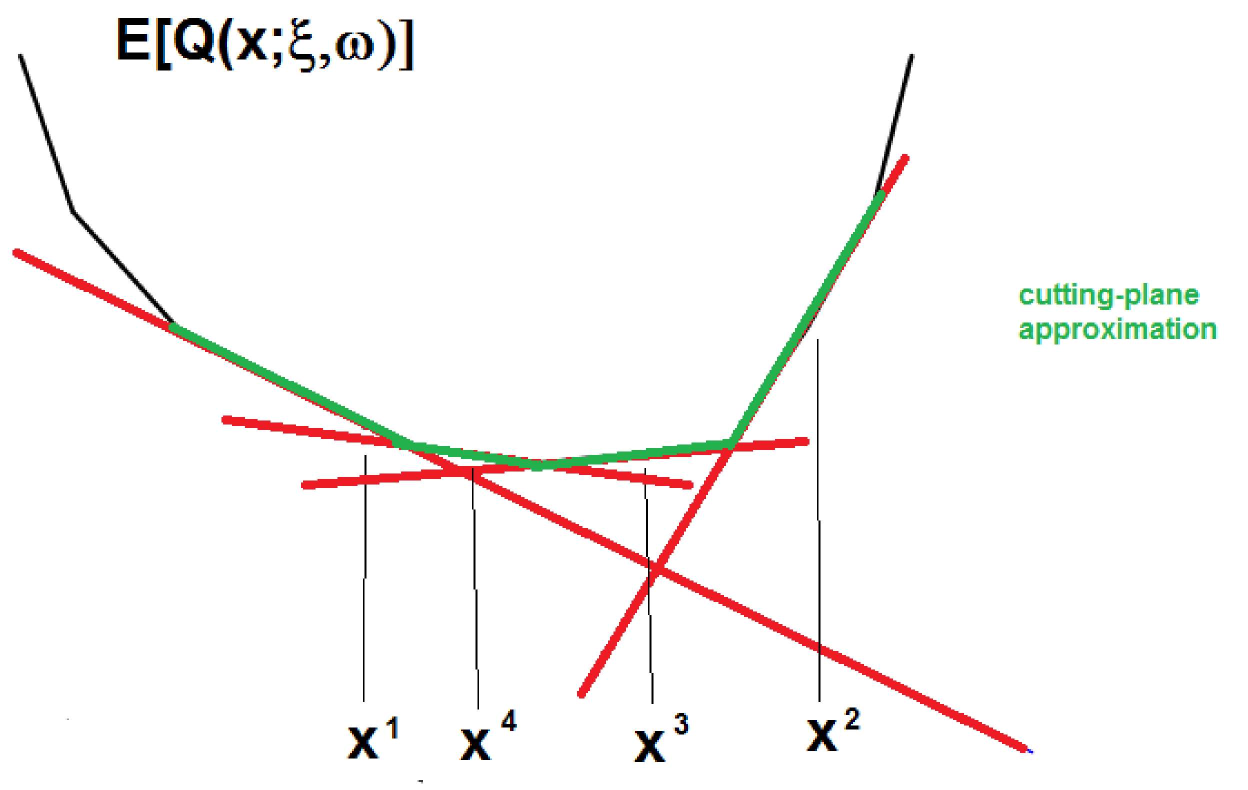
Cutting-plane approximation

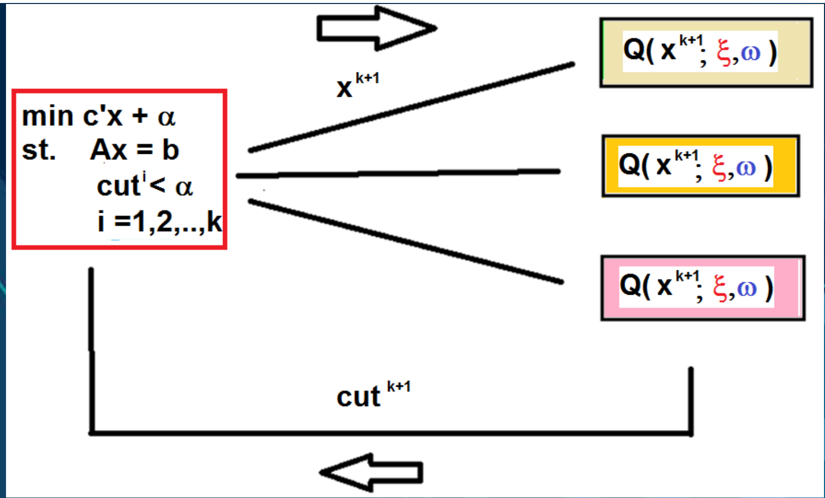


Cutting-plane approximation



Cutting-plane approximation





- N linear programming problems must be solved for every first-stage decision x^k

$$Q(x;\xi,\omega)\begin{cases}\min\limits_{y} & q(\xi)^\top y \\ \text{s.t} & Wy \quad = h - Tx \\ & \underline{y} \leq \quad y \quad \leq \bar{y} \\ & y_i \quad = \bar{y}_i(\omega) \quad i \in I_V\end{cases}$$

- This is a difficult task for large values of N
  - We may solve the LPs in a approximate manner (inexact cuts)
  - Use more efficient cutting-plane methods, such as Bundle Methods:

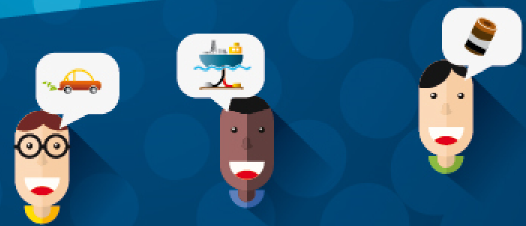
Convex proximal bundle methods in depth: a unified analysis for inexact oracles.
Math. Programming, 2014, 148, 1-2, pp 241-277
W. de Oliveira, C Sagastizábal and C. Lemaréchal.

Agenda

1. Project Team
2. Brazilian Oil Supply Chain
3. Current Approach
4. Uncertain Sources
5. Proposed Approach
    a. Statistical Model
    b. Optimization Model
6. Conclusions

Conclusions

- Stochastic PlanAb
  - Price scenarios
  - Oil volume
  - can be modelled either by using scenarios or chance-constraints
  - follow independent log-logistic probability distributions
- The computational implementation of the stochastic PlanAb model, with price scenarios and chance-constraints for oil volumes is in progress
- Challenge of modeling and computational performance
- Due to the problem of features there are enormous potential gain



Stochastic optimization models for short-term oil production

The team

Tito Homem-de-Mello
Universidad Adolfo Ibañez

Bernardo Pagnoncelli
Universidad Adolfo Ibañez

Davi Valladão
PUC-Rio

Thuener Silva
Tecgraf

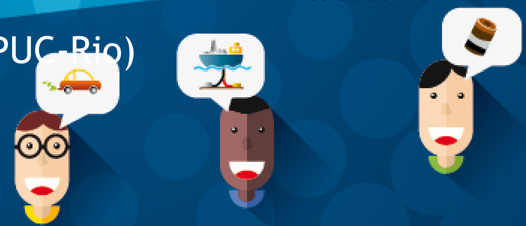
Carlos Gamboa
PUC-Rio

Bruno Vieira
Petrobrás

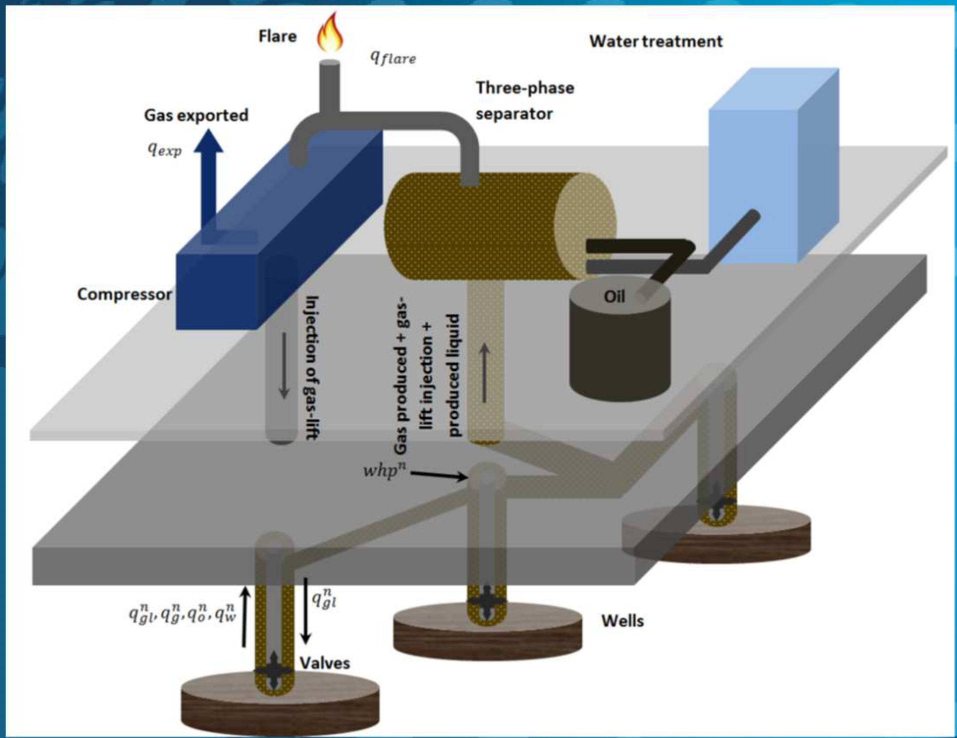
Alex Teixeira
Petrobrás

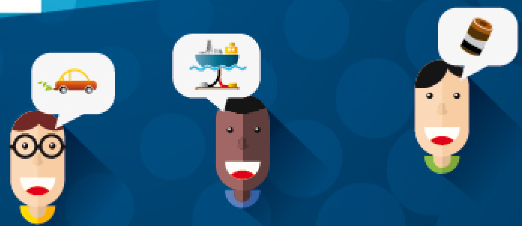
Vinicius Rosa
Petrobrás

- This work is part of a larger project between Petrobras and Tecgraf (PUC-Rio)

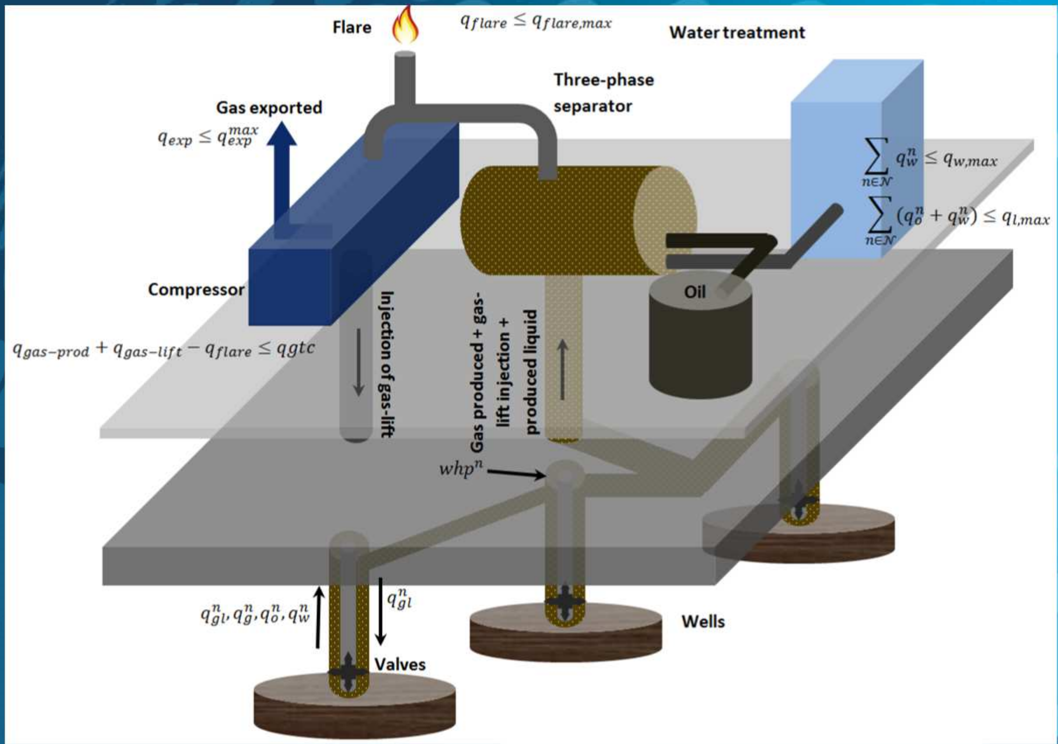


The OIL production process





The OIL production process

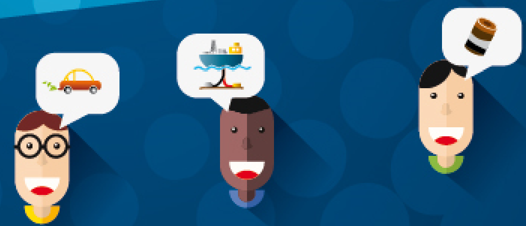


The problem

- For a system in steady-state, find the optimal values of
  - Gas-lift
  - Wellhead pressure
  - Well opening and closing (O/C)

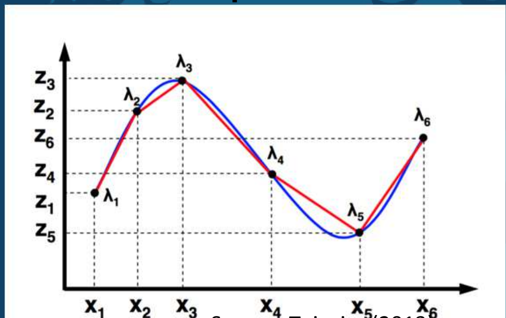
  that optimize the production.

- The in-house software Marlim is used to simulate the oil production for given values of the gas-lift and wellhead pressure.



Current status

- Petrobras currently uses the model BR-SiOP (Teixeira 2013) to solve this problem.
  - The model that computes the values of gas-lift, wellhead pressure and well O/C that optimize the production, subject to constraints on capacity of fluid treatment.
  - The output from Marlim is modeled as a piecewise linear

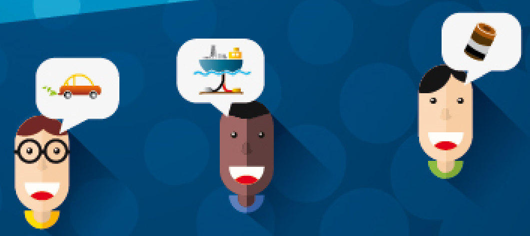


Source: Teixeira (2013)

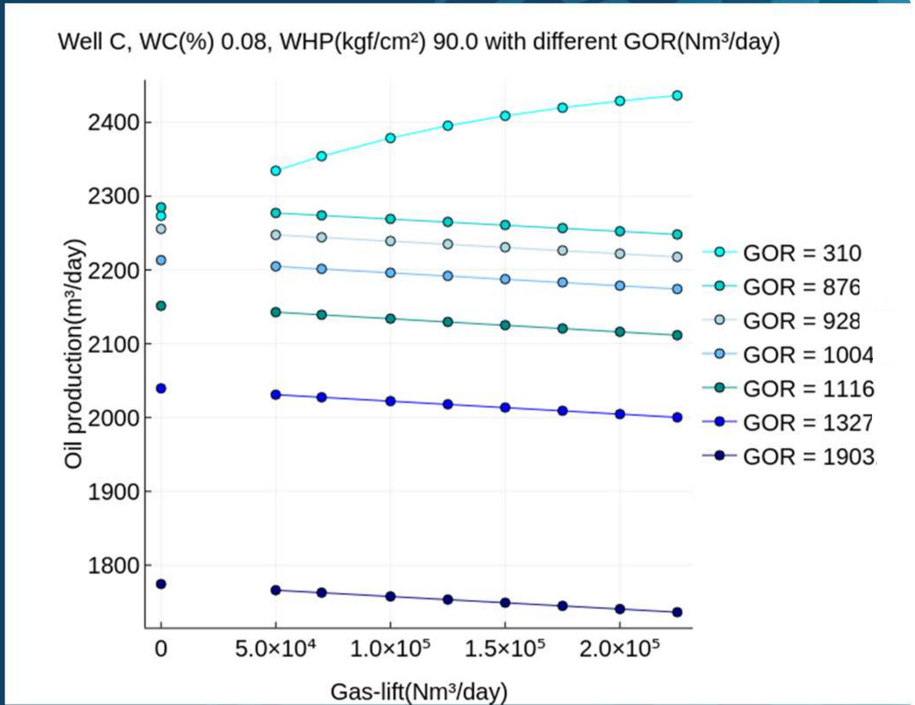
By using binary variables to represent the piecewise linear approximation, the resulting model is a <u>mixed linear integer optimization</u> problem which can be solved with commercial optimization software.

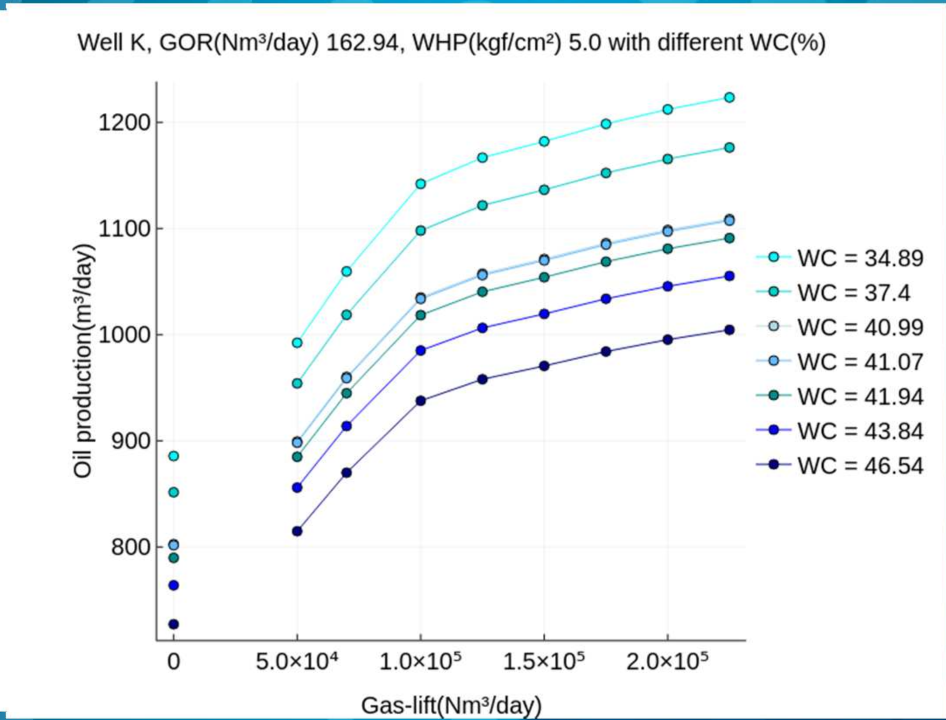
Shortcomings of br-siop

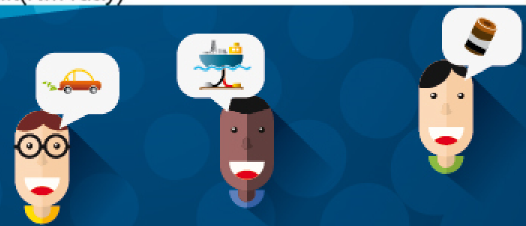
- While BR-SiOP has had significant impact on the oil production process, it has some drawbacks:
  - It does not consider randomness of some important parameters of the process such as the Gas-Oil Ratio (GOR) and the Water Cut (WC).
  - The use of binary variables to model the oil production curves hampers the extension to larger cases – for instance, if one wants to consider different scenarios of GOR and WC.



IMPACT OF GOR AND WC ON PRODUCTION



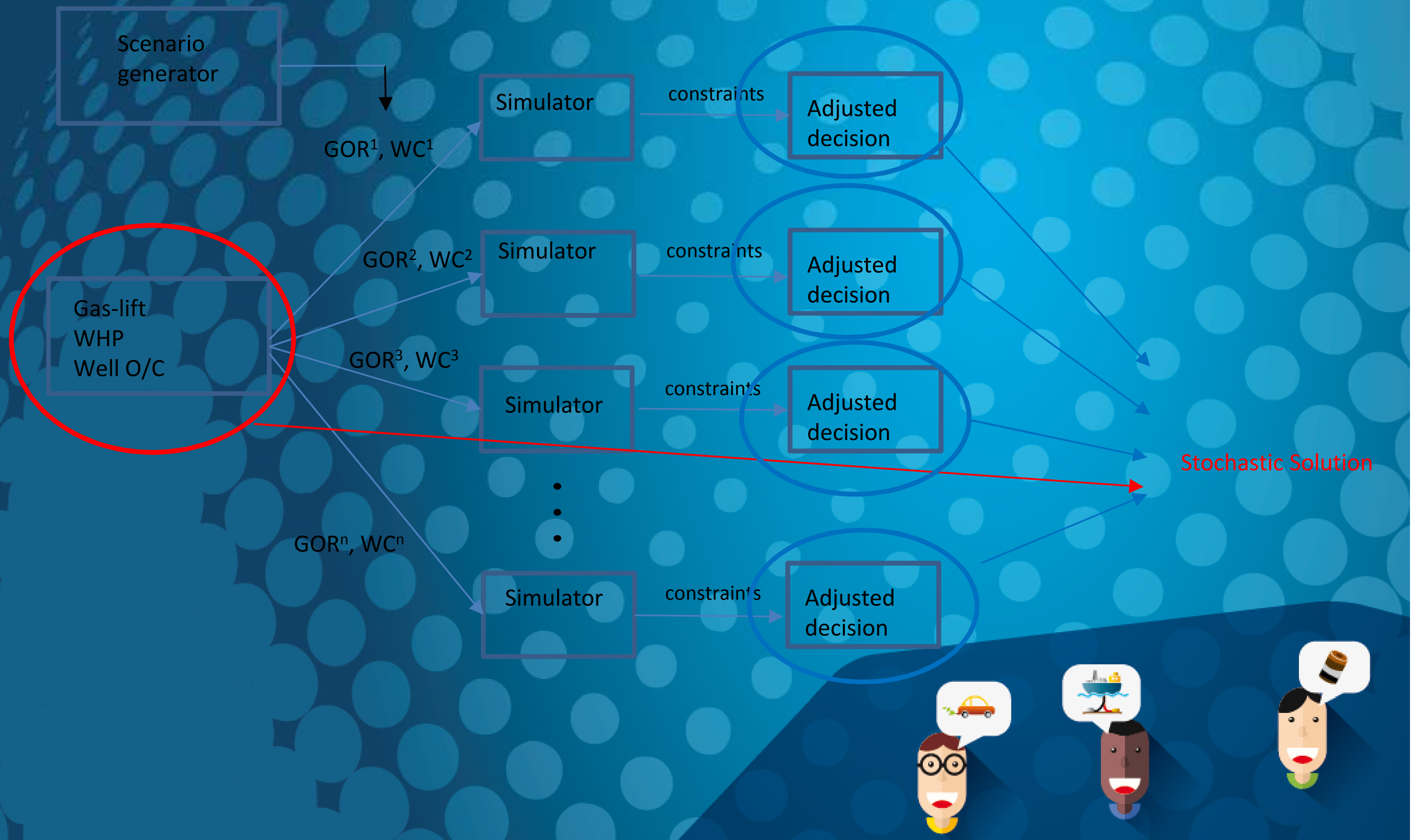




A stochastic optimization model

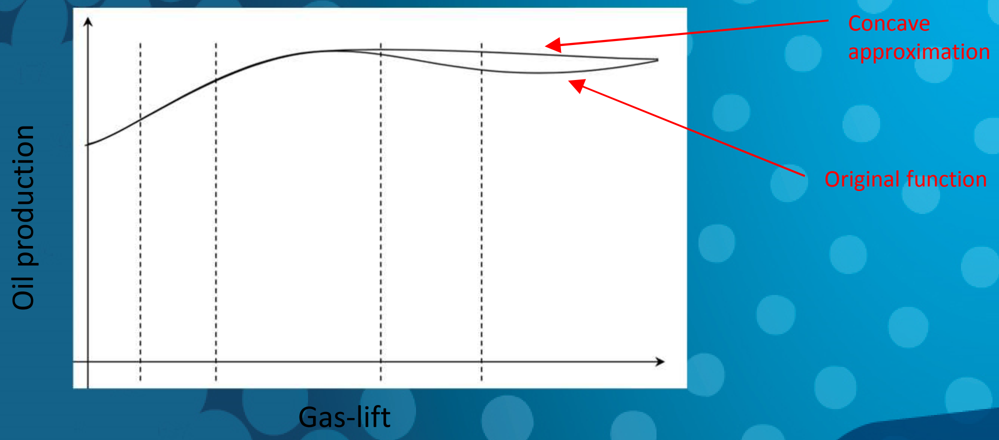
- We propose a two-stage stochastic optimization model for the problem.
- The main idea is to provide baseline values of gas-lift, wellhead pressure and well O/C that protect against the variations in GOR and WC, represented by scenarios.
- The second stage of the problem allows for controlled deviations from the baseline values of gas-lift.

Schematic representation



Concave approximations of production function

- As the complexity of solving the problem grows with the numbers of scenarios, we consider concave approximations of the production curve that do not require (millions of) integer variables.
- Idea: points at non-concave regions are unlikely to be optimal.



The model

- Objective function:

$$\max_{\mathbf{q}_{gl}^1,\mathbf{whp},\mathbf{t}} \frac{1}{\Omega}\sum_{\omega\in\Omega} V(\underbrace{\mathbf{q}_{gl}^1,\mathbf{whp},\mathbf{t}},\omega)$$

The parameter ω indicates a scenario of GOR, WC

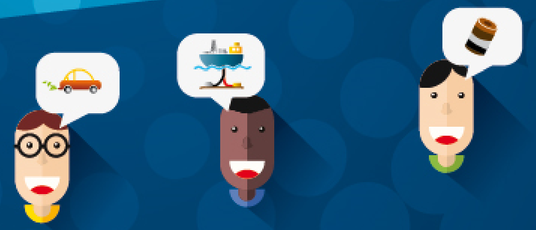
Decision variables (each variable is a vector, one value per well)

where

$$V(\mathbf{q}_{gl}^1,\mathbf{whp},\mathbf{t},\omega) = \max \sum_{n\in\mathcal{N}} q_o^n(\omega) + \alpha \cdot q_{exp}(\omega)$$

Oil production for well *n* in scenario ω

Exported gas in scenario ω



The model

- Constraints (2nd stage):
  - Oil production: $q_o^n(\omega) \leq \hat{q}\left(q_{gl}^{2,n}(\omega), whp^n, \omega\right) \qquad \forall n \in \mathcal{N}.$
  - Linking constraints:

$$q_{gas-prod}(\omega) = \sum_{n \in \mathcal{N}} q_g^n(\omega),$$

$$q_{gas-lift}(\omega) = \sum_{n \in \mathcal{N}} q_{gl}^n(\omega),$$

$$q_{exp}(\omega) = q_{gas-prod}(\omega) - q_{flare}(\omega) + q_{turbine}.$$

  - Capacity constraints on compression, flare, exportation, and water and liquid treatment.
  - GOR, WC defining constraints:

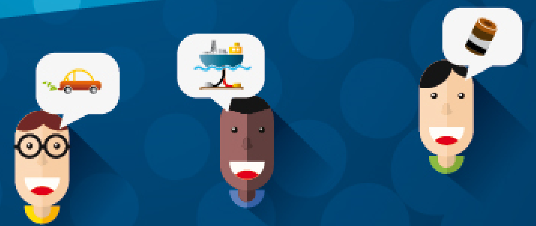
$$q_g^n(\omega) = GOR^n(\omega) \cdot q_o(\omega), \quad \forall n \in \mathcal{N}.$$

$$q_w^n(\omega) = \frac{WC^n(\omega)}{1 - WC^n(\omega)} \cdot q_o(\omega), \quad \forall n \in \mathcal{N}.$$

  - Well opening/closing: $t^n \cdot q_{gl,min}^n \leq q_{gl}^{2,n}(\omega) \leq t^n \cdot q_{gl,max}^n \quad \forall n \in \mathcal{N}.$
  - Maximum deviation from baseline: $\left|q_{gl}^{1,n} - q_{gl}^{2,n}(\omega)\right| \leq \Gamma \cdot q_{gl}^{1,n}, \ \forall n \in \mathcal{N}.$

Concave approximation of production function

The user-defined parameter Γ controls the maximum deviation (in %) from the baseline values

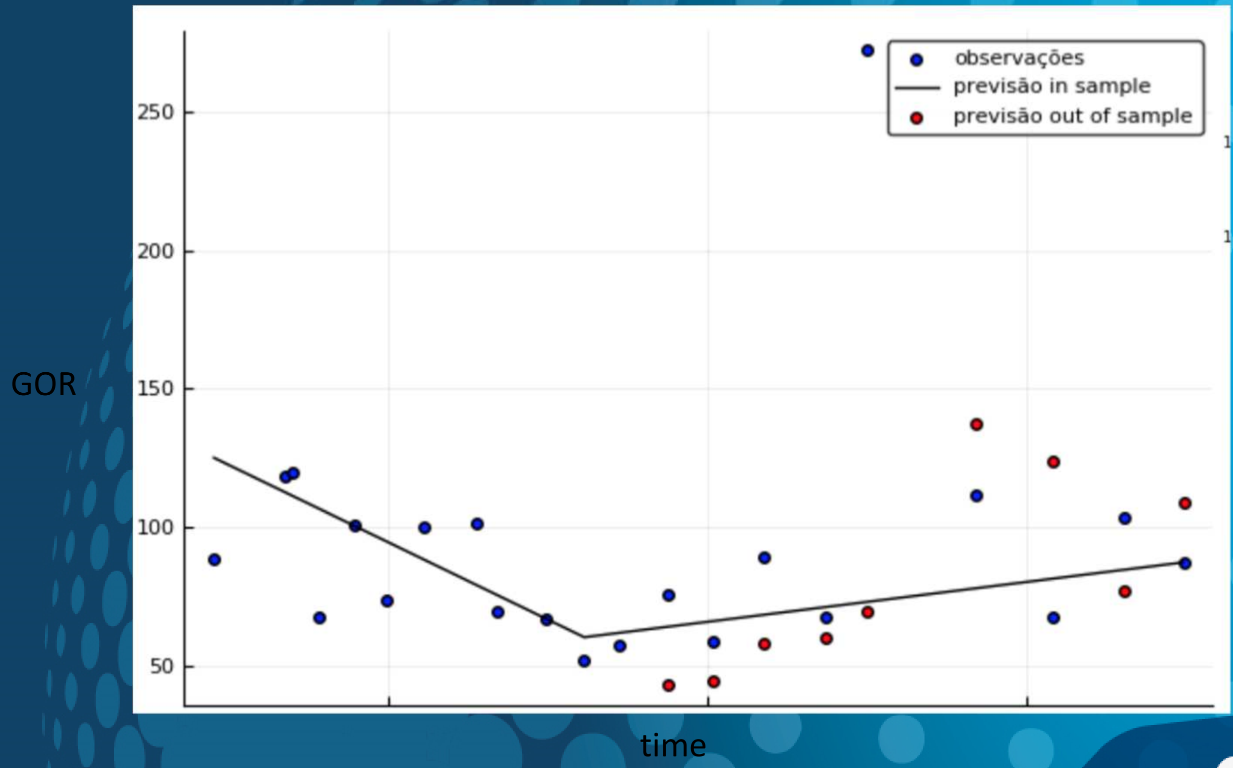


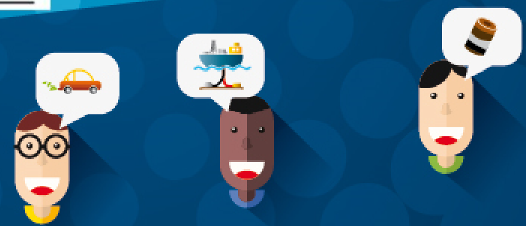
Generating scenarios

- The stochastic optimization model requires scenarios of GOR and WC.

- We used past measurements to forecast future values of these parameters using L1-filters, and then solved the problem for sampled scenarios.
  - This procedure is called Sample Average Approximation (Kleywegt et al. 2002).
  - We also applied techniques from the literature to determine an appropriate number of sampled scenarios.

L1 filters: illustration





results

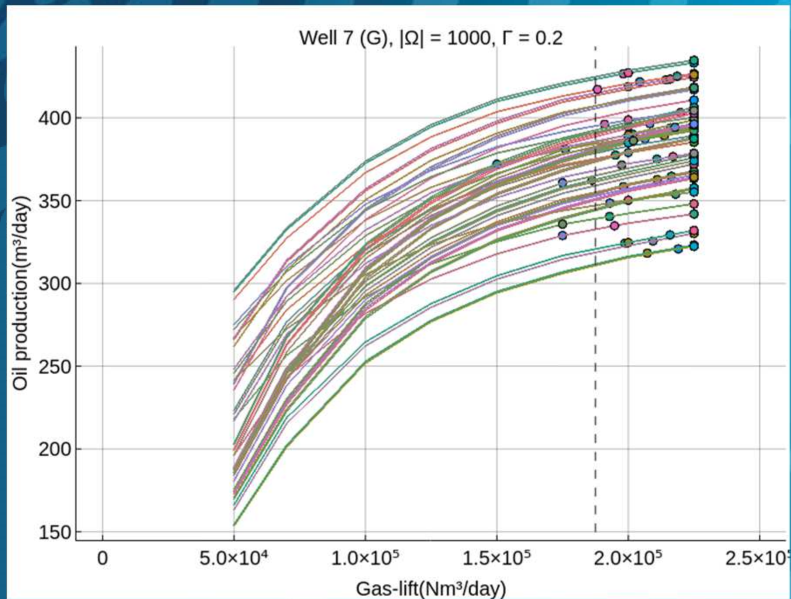
- We tested our model with real data of a platform with 11 wells.
- The distribution of GOR and WC was assumed to be independent across wells.
  - If correlations among wells can be reliably estimated, it is easy to generate correlated samples.
- We considered 4 values for the deviation parameter Γ (0%, 5%, 10%, 20%).
- We used 1,000 scenarios of GOR and WC.
- Code was implemented in Julia/JuMP.

Optimal solution,

	Compression Capacity								
	33.3%			66.6%			100%		
	RP	VSS	(%)	RP	VSS	(%)	RP	VSS	(%)
0.0	3121	70.2	2.25	4952	0.4	0.01	6213	11.5	0.19
0.05	3148	65.4	2.08	4965	3.2	0.06	6232	15.5	0.25
0.2	3193	38.4	1.20	4986	1.1	0.02	6257	5.5	0.09

OPTIMAL PRODUCTION CURVES

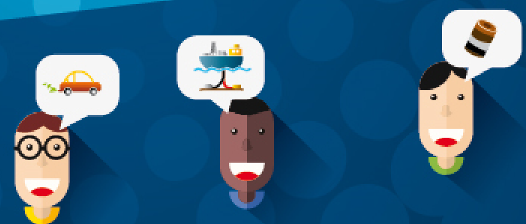
- Production curves per scenario of GOR, WC for one of the wells



Optimal values of gas-lift for each scenario are marked with a circle.

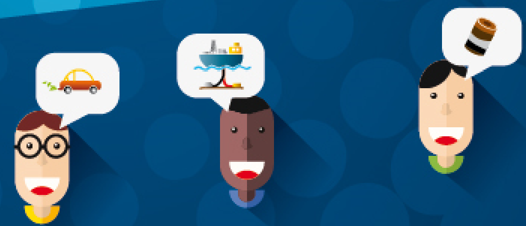
The dashed line corresponds to the baseline solution.

The curves are plotted for the optimal value of wellhead pressure.



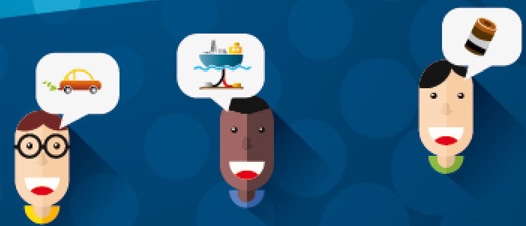
Comparison with the deterministic model

- We compared our model with a deterministic model that uses the average values of GOR and WC.
- The comparison is made by
  1. Computing the objective function value of the two-stage model at the solution given by the deterministic model, and
  2. Comparing it with the optimal objective value of the two-stage model.
- Some observations:
  - When the deterministic solution is feasible, the difference is small.
  - However, the deterministic solution is often infeasible for some scenarios due to capacity constraints– for instance, when either GOR or WC are high.
  - This suggests that the one of the benefits of the stochastic model is the adaptation of the solution to the observed scenarios of GOR and WC.



Conclusions

- We have presented a model for oil production that takes into account uncertainties in the composition of the oil such as the gas-oil ratio and water cut.
- The model produces a baseline solution as well as a solution adapted to each scenario of uncertainty that does not deviate too much from the baseline solution.
- The adaptation to scenarios ensures feasibility of the solutions.
- Future work includes, among other tasks:
  - The incorporation of uncertainty of reservoir parameters.
  - Integration with other forecast methods for GOR and WC.
  - Implementation of the strategy suggested by the model.



THANK YOU

And thanks to PETROBRAS and EMBRAPIII