From theory to practice: challenges in providing stochastic optimization tools for the energy planning of real systems

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OUTLINE

- Stochastic programming/optimization tools for expansion and operation planning of the large-scale Brazilian system

- **Challenge #1:** Handling risk averse problems

- **Challenge #2:** Sampling backward SDDP scenarios

- **Challenge (aspect) #3:** Resampling forward SDDP scenarios

- **Challenge #4:** Trade-off between system Representation and quality of the results

- **Challenge #5:** Performance requirements

- **Challenge #6:** Handling high uncertainty/variability of intermittent sources

- **Bonus:** Validation of DESSEM model for hourly prices in Brazil
Stochastic programming / optimization tools for expansion and operation planning of the large-scale Brazilian system
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BRAZILIAN INTERCONNECTED SYSTEM (SIN)

GENERATION MIX – 2016 & 2021

source: www.ons.org.br
MAIN CHARACTERISTICS OF THE BRAZILIAN SYSTEM

- Large-scale system, predominantly hydro
- Stochastic inflows to reservoirs
- Long distances between generation sources and load
- Many hydro plants in cascade

Coordinated operation is a VERY complex task!
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HYDROTHERMAL PLANNING FOR THE BRAZILIAN INTERCONNECTED SYSTEM

Developed by CEPEL, collaborating with scientific community

Validated in working groups in by ONS, CCEE, EPE, MME, ANEEL, as well as task forces with most power system utilities

Approved for official use by the regulatory energy

Used by:
- EPE to plan the system
- ONS to dispatch plants
- CCEE for market prices

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**NEWAVE**

- Long, Mid and Short Term Generation Planning
- MILP
- Application of CVaR Risk Averse Mechanism

**SDDP**

- [Pereira, Pinto, 91]
- [Maceira, 93]

**DECOMP**

- [Birge, 85]
- MultiStage Benders Decomposition

**DESEMM**

- [Shapiro, Tekaya, Costa, Soares, 12]
- [Diniz, Tcheou, Maceira, 12]

**HYDROTHERMAL PLANNING FOR THE BRAZILIAN INTERCONNECTED SYSTEM**

- Long, Mid and Short Term Generation Planning
- NEWAVE
- SDDP
- DECOMP
- DESSEM
- MILP
- Application of CVaR Risk Averse Mechanism

References:

- [Maceira, Duarte, Penna, Mor, Mel, 08]
- [Maceira, Penna, Diniz, Pinto, Melo, Vasconcel., Cruz, 18]
- [Diniz, Costa, Maceira, Santos, Brandao, Cabral, 18]
- [Diniz, Souza, Maceira et al, 02]
- [Diniz, Santos, Saboia, Maceira, 18]
MULTI-STAGE STOCHASTIC LINEAR PROGRAMMING PROBLEM

OBJECTIVE FUNCTION

$$\min \left[ \left( \sum_{t=1}^{T} \left( \lambda E + (1 - \lambda) CVar^\omega \right) \sum_{j=1}^{NT} c_j \left( GT_j^t, \omega \right) \right) + \alpha (v^T) \right]$$

Cvar Criterion
Thermal generation + load scheduling

CONSTRAINTS

Demand balance
$$\sum_{i \in H_j} gh_i^t, \omega + \sum_{i \in T_j} gt_i^t, \omega + \sum_{i \in Int_j} Int_i^t, \omega + Defc_j^t = D_j^t, \quad \forall t, \omega, j$$

Water conservation
$$v_s^t = v_s^{t-1} - gh_s^t, \omega + \xi_s^t, \omega \left| \left( \xi_s^{t-p, \omega}, p-l, ... p, \zeta \right) \right), \quad \forall t, \omega, j$$

Par-P model

+ Many other operation constraints...

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[Long Term Model – Newave Problem Formulation]

[Birge,Louveaux,97]
[Kall,Mayer,10]

[Selectiv Sampling]

[Maceira, Bezerra,97]

[Penna,Maceira,Damazio,11]
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**SDDP - STOCHASTIC DUAL DYNAMIC PROGRAMMING**

**FORWARD PASS**

- Scenario 1
- Scenario 2
- Scenario 3
- Scenario S << K^T

**BACKWARD PASS**

- Scenario 1
- Scenario 2
- Scenario 3
- Scenario S << K^T

Benders cuts:

\[ \varphi_t(x_{t-1}) \geq \sum_{\omega=1}^{K} p_{\omega} \left[ z_{t, \omega} + \langle \frac{\partial z_{t, \omega}}{\partial x_{t-1}} \rangle (\hat{x}_{t-1, s} - x_{t-1}) \right] \]

**OUTPUT:**

Optimal Operation Policy
MID-TERM MODEL - DECOMP

- Weekly steps for the 1st month
- Monthly steps for the following months, with several water inflow scenarios
- Several load blocks for each time step
- Coupling with long term model takes into account history of the system

Future cost function (FCF) provided by long term model (NEWAVE)
Challenge #1:
Handling risk-averse problems in large-scale systems
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CVaR RISK MEASURE APPLIED TO SDDP (2013)

APPLICATION TO MULTISTAGE HYDROTHERMAL PLANNING

[Shapiro,10]
[Philpott, de Matos,12]

Probability density function

Total cost of each scenario ($)

Wet scenarios

Dry scenarios

Scenario tree

\[ t = 0 \]

\[ 1 \quad \ldots \quad i \quad \ldots \quad T \]

\[ x_0 \]

\[ x_1^1 \]

\[ x_1^2 \]

\[ x_1^3 \]

\[ x_1^4 \]

\[ x_T^1 \]

\[ x_T^K \]

\[ CVaR(x)_{policy(*)} = \min_{pol} CVaR(x)^{pol} \]

Optimal risk-averse policy (*)

Other policies (k) (including the risk-neutral policy)
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DIRECT APPLICATION OF NESTED CVAR RISK- AVERSE CRITERION

**BACKWARD PASS**

- solve subproblems for all backward scenarios \( \omega \)
- identify the scenarios related to the \( \alpha \)% highest values of \( z_{t,\omega} \)
- Build Benders cuts with both expected value and risk averse terms

\[
\varphi_t(x_{t-1}) \geq (1-\lambda) \sum_{\omega=1}^{K} p_\omega \left[ Z_{t,\omega}^* \left( \pi_{t,\omega}^* \cdot x_{t-1} - \hat{x}_{t-1,s} \right) \right] \\
+ \left( \frac{\lambda}{\alpha} \right) \sum_{\omega \in \Omega_\alpha} p_\omega \left[ Z_{t,\omega}^* \left( \pi_{t,\omega}^* \cdot x_{t-1} - \hat{x}_{t-1,s} \right) \right] = \bar{Z}^* + \left( \pi^* , x_{t-1} - \hat{x}_{t-1,s} \right)
\]
Explicit protection against critical scenarios

- Feasibility cuts (SAR constraints)
- Deterministic subproblem (critical scenario, several months ahead)
- Benders Decomposition 2 stages

Scenarios in the NEWAVE model

- Shared cuts
- Storages at the end of time step $t$

Novel penalization scheme to avoid a large increase in system marginal costs => Maximum violation along each year

- Explicit protection against critical scenarios

- [PSR, 08]
- [Diniz, Maceira, Vasconc., Penna, 14]

- [Diniz, Maceira, Vasconc., Penna, 16]
Yielding the **least cost is not anymore a criterion** to select the most suitable policy

A multicriterion analysis has to be conducted making an out-of-the sample assessment for different hydrological and system conditions of many aspects such as:

- **Thermal generation costs**
- **X deficit risk (costs)**
- Distribution of energy not served (ENS)
- Probability of spillage
- Evolution of the storage of the reservoirs

[Maceira, Marzano, Penna, Diniz, Justino, 2014]
Challenge #2: Sampling backward SDDP scenarios
SOME NOTES ON BACKWARD SCENARIO GENERATION

- In sample average approximation (SAA) algorithms, usually a random (Monte Carlo) sampling is recommended to generate scenarios, in order to have more diversity
  - Many replications of the scenario tree should be performed

- However, **the problem can only be solved ONCE**

- The use of random sampling in backward scenarios makes the results **very sensitive to the seed** used to generate scenarios

Therefore, it is recommended to use clustering techniques when generating backward scenarios, even though the value of the optimal solution may be (a bit) biased.

Also, while clustering, picking the centroid as representative of each cluster (instead of the closest object to the centroid) brings more stable results.
Inflow values at each time step, for different seeds

**MONTE CARLO**

- Current option

**SELECTIVE SAMPLING (2009) (K-MEANS)**

- Selective Sampling (SS)

[Penna, Maceira, Damazio, 11]
Lower bound for the optimal solution
Challenge (aspect) #3: Resampling Forward SDDP scenarios
OBJECTIVE: To allow a more representative part of the HUGE multi-stage scenario tree to be visited.

SDDP convergence has been proved once forward resampling is performed.

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RESAMPLING - RESULTS FOR DIFFERENT SEEDS (1/2)

LOWER BOUND FOR EACH SEED

No resampling

FULL RESAMPLING

RECOMBINING SAMPLING

k = 1

Resampling step (each k iterations)
The main objective of NEWAVE model is **not to solve the discrete mathematical problem**, but rather to **obtain an operation policy** for the **continuous problem**.
The main objective of NEWAVE model is **NOT TO SOLVE THE DISCRETE MATHEMATICAL PROBLEM**, but rather to **OBTAIN AN OPERATION POLICY** for the **CONTINUOUS PROBLEM**

**Our model of the hydro inflows for t=1**

**exact FCF for t=1 of the mathematical problem (depends on green scenarios in t=2)**
The main objective of NEWAVE model is **not to solve the discrete mathematical problem**, but rather to **obtain an operation policy** for the **continuous problem**.

Our backward sample of the hydro inflows for $t=1$

The exact FCF for $t=1$ of the *mathematical* problem (depends on green scenarios in $t=2$)
The main objective of NEWAVE model is **NOT TO SOLVE THE DISCRETE MATHEMATICAL PROBLEM**, but rather to **OBTAIN AN OPERATION POLICY** for the **CONTINUOUS PROBLEM**.

- **Approximation of the FCF if forward samples are taken from the continuous distribution**
- **Exact FCF for t=1 of the mathematical problem (depends on green scenarios in t=2)**

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The main objective of NEWAVE model is **not to solve the discrete mathematical problem**, but rather to **obtain an operation policy** for the **continuous problem**.

**Exact FCF for t=1 of the mathematical problem** (depends on green scenarios in t=2)

- Approximation of the FCF if forward samples are taken from the continuous distribution
- Approximation of the FCF if forward samples are taken from backward scenarios (much smaller number of visited states)
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Which one is the best policy for $t=1$?

- The red points are enough to find the optimal solution for the **mathematical problem** (red + green scenarios).
- However, we know this is NOT the true problem: we should be prepared for the “real” **continuous distribution**.
- So, why do we use a discrete backward distribution?
  - We must set a tractable and exact mathematical problem.

Exact FCF for $t=1$ of the **mathematical problem** (depends on green scenarios in $t=2$)

Approximation of the FCF if forward samples are taken from the continuous distribution

Approximation of the FCF if forward samples are taken from backward scenarios (much smaller number of visited states)
The main objective of NEWAVE model is not to solve the problem, but rather to obtain an operation policy.

- With such policy it is possible to simulate a large number of scenarios in order to obtain proper statistics for system operation:
  - Average System Marginal costs
  - Average thermal generation
  - Average storage in the reservoirs along time
  - Deficit risk and average load curtailment
  - Average spillage

- The policy should be “good” for **any set of scenarios in the continuous distribution** of the random variable.

Therefore, for practical applications, we sample the forward scenarios from the continuous distribution, rather than the set of backward noises, even though this is a theoretical requirement to find the optimal solution of the mathematical problem (our aim is NOT to find it!!)
Challenge #4: Trade-off between system representation and quality of the results
MODELING OF EQUIVALENT RESERVOIRS IN THE LONG TERM

- Individual aspects are modeled as much as possible
  - Loss of efficiency with the water head
  - Inflows/spillage in run of the river plants, etc.

- System Areas
  - EERs
  - 4 System Areas

- Thermal plants
  - tie lines
  - hydro coupling
The main goal is to capture diversity of the hydrological behaviour, while still keeping a lower state space.

Due to market aspects, the number of market areas is kept.

The increase in the number of EERs is based on comprehensive studies performed by CPAMP (ONS, CCEE, EPE, MME, ANEEL, CEPEL)

- until 2015: **4 EERs**
- 2016: **9 EERs**
- 2018: **12 EERs**

**[de Matos, Finardi, Silva, 08]**

**[Ennes, Penna, Maceira, Diniz, Vasconcellos,11]**
HYBRID INDIVIDUAL / EER REPRESENTATION IN NEWAVE

- Allows NEWAVE model to represent the hydropower plants individually in the entire or in part of its planning horizon

- Takes advantage of both modellings, without increasing too much the computational effort by considering:
  - the benefits of an individual representation of HPPs in the horizon closer to the operational decision making
  - as many EERs as necessary to represent the hydrological diversity among the river basins, in the later stages
SOME NOTES ON SYSTEM REPRESENTATION

- From the system modeling point of view, the consideration of individual reservoirs in SDDP itself is not a challenge
  - Constraints are similar (and simpler) than mid/short term models, and the construction of Benders cut in DDP or SDDP is similar

- Modeling of random variables (e.g. past inflows) for individual plants is more involving, due to its high dimensionality and the statistical model (overparametrization, spatial/serial correlations...)

- One major aspect is the trade-off between system representation and the quality of the results
  - It is not enough to simply run a number of SDDP iterations: it is VERY important to make sensitivity analysis on the final simulation results

- A critical issue is the quality and impact of the detailed data related to system components and constraints in the long term
  - It is not possible to satisfy all constraints in all future scenarios: in practice, some constraints are adjusted according to the system state, and one cannot simply discard violated scenarios
  - Aggregate constraints may be more effective in the long run
Challenge #5: Performance Requirements
COMPUTATIONAL EFFICIENCY AND QUALITY OF THE RESULTS

Parallel processing

Solving strategy for economic dispatch subproblems
- warm starts
- optimal simplex basis recovery (LPs)
- dynamic piecewise linear models
- cut selection
- MIP: Local branching

Reproducibility of the results
- The results should be identical regardless of the machine and number of parallel processors

Strict validation process
- CPAMP committee
- Task forces for each model
- + 300 users

Programs are heavily tested
Challenge #6:

Handling high uncertainty/variability of intermittent sources
Intermittent renewable generation turns the traditional deterministic unit commitment model into a stochastic unit commitment model.

- Wind generation has to be represented as interruptible generation, for feasible AND economic reasons.

- Reliable models for scenario generation of wind production should be developed.

[Cotia, Borges, Diniz, 19]

[Pessanha et al, 18]
"SOFT LINK"

- integration between long term and short term models

"HARD LINK"

- directly represent hourly operation constraints in the long term model

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[Deane, Chiodi, Gargiulo, 12]  
[Deane, Chiodi, Gargiulo, 12]  
[Tejada-arango Domeshek, Wogrin 18]  
[Deane, Chiodi, Gargiulo, 12]  
[Deane, Chiodi, Gargiulo, 12]  
[Pina, Silva, Ferrão, 13]  
[Pina, Silva, Ferrão, 13]  
[Tejada-arango Domeshek, Wogrin 18]
This list of challenges is BY FAR not exhaustive:

- How to make SDDiP practical?
- How to better handle long term uncertainties? (e.g. load growth)
- How to better address short range uncertainties on intermittent generation?
- Consideration of climate changes
- etc, etc, etc.
REFERENCES


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OBRIGADO!

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Hourly prices with DESSEEM model
Validation of DESSEM model for hourly prices in Brazil
DETAILED REPRESENTATION OF THE ELECTRICAL NETWORK

[From theory to practice: challenges in providing stochastic optimization tools for the energy planning of real systems]

Half-an-hour / hourly intervals

1-2 days

remaining days

Larger intervals

DECOMP FCF

[Diniz, Santos, Saboia, Maceira, 18]

System Areas

Hydro Plants

Thermal Plants

Power injections

loads

Interchanges among areas

Transmission lines

River courses

N

NE

SE

S

IT

IV

B

remaining days

Half-an-hour / hourly intervals

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[Diniz, Santos, Saboia, Maceira, 18]
HYDRO PLANTS
PRODUCTION FUNCTION

VARIATION OF EFFICIENCY WITH THE WATER HEAD

\[ GH_i = 9.81 \times 10^{-3} \eta Q \left( h_{up}(V) - h_{dn}(Q + S) - h_{loss} \right) \]

Example of HPF as a function of \( V \) and \( Q \)

Example of HPF as a function of \( S \) (fixed values of \( V \) and \( Q \))

Four-dimensional piecewise linear model
[Diniz, Maceira, 08]
HYDRO CONSTRAINTS

HYDRO PLANTS IN CASCADE

- Modeling of river sections
- Evaporation in reservoirs
- Multiple uses of water
- Flood control constraints
- Filling of "dead volumes"
- Channels between reservoirs
- Pumping stations
- Minimum releases

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RIVER ROUTING

- Representation of water propagation curves along the river courses

$$V_i^t + (Q_i^t + S_i^t) - \sum_{j \in M} \sum_{k=0}^{t_i} k_{ij,k}(Q_{ij,k}^t + S_{ij,k}^t) = V_{i-1}^t + I_i^t$$
NONCONVEX THERMAL UNIT COMMITMENT CONSTRAINTS (1/2)

Minimum generation (once ON)
\[ u_i^t \in \{0, 1\} \quad gt_i \cdot u_i^t \leq gt_i^t \leq gt_i \cdot u_i^t \]

Startup/shutdown costs
\[
\begin{align*}
C_{cold} & \quad Ce_i^{t}(u_i^t - \sum_{k=t-\tau}^{t} u_k^k) \leq S_i^t \\
C_{hot} & 
\end{align*}
\]

Ramp constraints
\[
\Delta gt^- \leq gt_i^t - gt_i^{t-1} \leq \Delta gt^+
\]

Minimum up/down times
\[
\begin{align*}
\sum_{k=t}^{t+Ton_i-1} u_i^k & \geq Ton_i \cdot (u_i^t - u_i^{t-1}) \\
\sum_{k=t}^{t+Toff_i-1} (1-u_i^k) & \geq Toff_i \cdot (u_i^{t-1} - u_i^t)
\end{align*}
\]
Start-up / Shutdown trajectories

\[ GT_i^t \geq GT_i \left( u_i^t - \sum_{k=1}^{NU_i} \hat{y}_i^{t-k+1} - \sum_{k=1}^{ND_i} \tilde{y}_i^{t+k-1} \right) + \sum_{k=1}^{NU_i} TrUp_i(k) \cdot \hat{y}_i^{t-k+1} + \sum_{k=1}^{ND_i} TrDn_i(ND_i - k + 1) \cdot \tilde{y}_i^{t+k-1} \]

\[ GT_i^t \leq GT_i \left( u_i^t - \sum_{k=1}^{NU_i} \hat{y}_i^{t-k+1} - \sum_{k=1}^{ND_i} \tilde{y}_i^{t+k-1} \right) + \sum_{k=1}^{NU_i} TrUp_i(k) \cdot \hat{y}_i^{t-k+1} + \sum_{k=1}^{ND_i} TrDn_i(ND_i - k + 1) \cdot \tilde{y}_i^{t+k-1} \]

Auxiliary variables:

\[ \tilde{y}_i^t = \tilde{w}_i^t + (u_i^{t-1} - u_i^t) \]

\[ \tilde{y}_i^t + \tilde{w}_i^t \leq 1 \]

\[ NU_i \]: length of startup trajectory

\[ ND_i \]: length of shutdown trajectory
MODELING OF COMBINED CYCLE PLANTS

- Application of a hybrid component/mode model
- Constraints can be individually enforced for the thermal units
- Transition requirements between configurations can be included
- Linking constraint between units status and configuration modes

\[
\begin{align*}
\sum_{j \in NU_i} P_j u_{ij} &= \sum_{k \in NC_i} N_k x_{ik}^t \\
\sum_{k \in NC_i} x_{ik}^t &= 1 \\
u_{ij}, x_{ik}^t &\in \{0,1\}
\end{align*}
\]

[Liu, Shahidehpour, Li, Mahmoud, 09] [Morales-Espana, Correa-Posada, Ramod, 16]
Line Flow limit constraints

\[ f_{km}^{t} \leq f_{km}^{t} = \frac{\theta_{k}^{t} - \theta_{m}^{t}}{x_{km}} \leq f_{km} \]

- phase angles are a function of power injections / loads

Ramp constraint on line flows

\[ |f_{km}^{t} - f_{km}^{t-1}| \leq \Delta f_{km} \]

Power transmission losses

- dynamic piecewise linear approximations

\[ l_{i} \approx g_{i} \Delta \theta_{i}^{(k)}^{2} \]

- Line flow limits and approximations for losses are iteratively included
constraints on the relation of flows in given lines of the system for security purposes

some constraints are given by tables

PIECEWISE LINEAR CONSTRAINTS

HEURISTIC ITERATIVE APPROACH
OVERALL PROBLEM FORMULATION

MIXED INTEGER (PIECEWISE) LINEAR PROGRAMMING

\[ \text{min } \sum_{t=1}^{T} \sum_{i=1}^{\text{NUT}} c_i (g_{ti}) + S_i^t + \alpha^T (V^T) \]

s.a.

\[ \sum_{j \in \Phi_i} g_{t+1}^j + \sum_{j \in \Psi_i} (\text{Int}_{t\rightarrow j} - \text{Int}_{t\rightarrow i}) = D_i^t \quad i = 1, \ldots, \text{NS}, \ t = 1, \ldots, T, \]

\[ \sum_{i=1}^{\text{NB}} \kappa_{ij} (g_{i} - d_{i}) \leq \overline{f}_i \ ; p_i - \sum_{i=1}^{\text{NB}} \kappa_{ij} (g_{i} - d_{i}) \leq \text{rhs} \quad i, j = 1, \ldots, \text{NL}, \ t = 1, \ldots, T \]

\[ V_i^t = V_{i-1}^t + I_i - (Q_i^t + S_i^t) + \sum_{j \in \Omega_i} (Q_j^t + S_j^t) \]

\[ gh_i^t = \text{FPH} (V_i^t, Q_i^t, S_i^t) \]

\[ \frac{V_i^t}{V_i^t} \leq V_i^t \leq \overline{V}_i^t, \ Q_i^t \leq Q_i^t \leq \overline{Q}_i^t, \ \frac{gh_i^t}{gh_i^t} \leq gh_i^t \leq \overline{gh}_i^t \]

\[ \text{QFT constraints: } \]

\[ gu^t_i \leq g_{ti} \leq \overline{g}_{ti} \cdot u^t_i \]

\[ \sum_{k=t}^{t+\text{Ton} - 1} u_i^k \geq \text{Ton}_i \cdot (u_i^t - u_i^{t-1}) \]

\[ \sum_{k=t}^{t+\text{Toff} - 1} (1 - u_i^k) \geq \text{Toff}_i \cdot (u_i^{t-1} - u_i^t) \]

\[ u_i^t \in \{0, 1\} \]

\[ \bar{y}_i^{t+1} = \bar{w}_i^t + (u_i^{t-1} - u_i^t) \]

\[ \bar{y}_i^{t+1} + \bar{w}_i^t \leq 1 \]

\[ C_{t+1} \left( u_i^t - \sum_{k=t}^{t-1} u_i^k \right) \leq S_i^t \]

\[ C_s \left( u_i^{t-1} - u_i^t \right) \leq S_i^t \]

\[ G_{T_i} \left( u_i^t - \sum_{k=1}^{\text{NU}_i} \hat{y}_i^{t-k+1} - \sum_{k=1}^{\text{ND}_i} \hat{y}_i^{t-k+1} \right) \leq \overline{G}_{T_i} \left( u_i^t - \sum_{k=1}^{\text{NU}_i} \hat{y}_i^{t-k+1} - \sum_{k=1}^{\text{ND}_i} \hat{y}_i^{t-k+1} \right) + \sum_{k=1}^{\text{NU}_i} \text{TrUp}_i(k) \cdot \hat{y}_i^{t-k+1} + \sum_{k=1}^{\text{ND}_i} \text{TrDn}_i(\text{ND}_i - k + 1) \cdot \hat{y}_i^{t-k+1} \]

\[ T \]

\[ H \]

\[ E \]

From theory to practice: challenges in providing stochastic optimization tools for the energy planning of real systems
Solving Strategy: MILP (1/3)

ITERATIVE LP APPROACH TO FIND MAJOR / POTENTIAL BINDING CONSTRAINTS IN THE ELECTRICAL NETWORK

1. Set the Multistage MILP Problem
2. Solve Linear Relaxation (LP)
3. Solve a DC load flow
4. Compute line flows
5. Insert line flow limits constraints
6. Satisfy Network constraints?
   - No: go back to step 3
   - Yes: continue...
7. Obtain a lower bound

References:
- [Stott, Marinho, 79]
- [Diniz, Souza, Maceira et al, 02]
- [Santos, Diniz, 11]
ITERATIVE LP WITH FIXED UC TO OBTAIN A GOOD (?) FEASIBLE SOLUTION

MILP solution (not feasible yet for the entire electrical network)

Fix the solution of integer variables

Solve LP (with a fixed UC)

solve a DC load flow compute line flows

Insert line flow limits constraints

Satisfy Network constraints?

Yes

Obtain an upper bound

Compute optimality gap

continue...

No

Optimal SIMPLEX basis

Obtained MILP solution (not feasible yet for the entire electrical network)
FINDING AN OPTIMAL SOLUTION WITH THE DESIRED ACCURACY

Feasible MILP solution

optimality criterion is met?

Yes

Publish optimal solution

No

Proceed solving the MILP problem
RECENT DEVELOPMENTS: REDUCTION OF CPU TIMES

- Use of tighter/more compact unit commitment formulations

- Taking into account optimal basis information while adding new constraints to the problem in a MILP setting

- Alternative (better) interaction between MILP and LP solving procedures
  - Application of local branching

Hamming Metric

\[ \Delta(u, \bar{u}) = \sum_{\{v: (\bar{u})^v = 1\}} [1 - (u)^v] + \sum_{\{v: (\bar{u})^v = 0\}} (u)^v \]

- [Ostrowsku, Anjos, Vanneli, 13]
- [Dam, Kucuk, Rajan, Atam, 13]
- [Morales-España, Latorre, Ramos, 13]
- [Ostrowsku, Anjos, Vanneli, 13]
- [Dam, Kucuk, Rajan, Atam, 13]
- [Morales-España, Latorre, Ramos, 13]
- [Fischetti, Lodi, 03]
- [Saboia, Diniz, 16]
DETERMINATION OF MARKET PRICES

Obtain nodal prices for all buses of the system

“Optimal solution has been reached” → Fix committent status of the units → Solve a continuous hydrothermal scheduling problem → Obtain multipliers for system area and line flow limits constraints

System are price as weighted average in all buses

Multipliers of demand balance in each area

\[
\sum_{i=1}^{NH,NT} g_i \pm \sum_{j \in \Omega_k} \text{Int}_{j \rightarrow i} = \sum_{i \in \text{Area } k} d_i, \quad k = 1,...,NS \quad \leftarrow \lambda^d_k
\]

Multipliers of line flow limits constraints

\[
\sum_{i=1}^{NB} \kappa_{il} g_i \leq f_{l \text{max}} + \sum_{i=1}^{NB} \kappa_{il} d_i, \quad l = 1,...,NL_{adic} \quad \leftarrow \lambda^f_l
\]

Nodal price:

\[
CMB_i = \lambda_{a(i)}^d + \sum_{l=1}^{NL_{adic}} \kappa_{il} \lambda^f_l
\]

System Area price:

\[
CMO_k = \frac{\sum_{i=1}^{NB} CMB_i d_i}{\sum_{i=1}^{NB} d_i}
\]
154 REAL CASES FROM Jan 1st to Jun 13th

Average CPU time: 18.2min
< 1 hour: ~ 96%

#Hydro Plants: 158
#Thermal Plants: 109
#Network Buses: 6,800
#Transmission Lines: 9,800