Robust Strategic Bidding in Day-Ahead Electricity Markets

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Workshop: Stochastic Programming Models and Algorithms for Energy Planning

Introduction

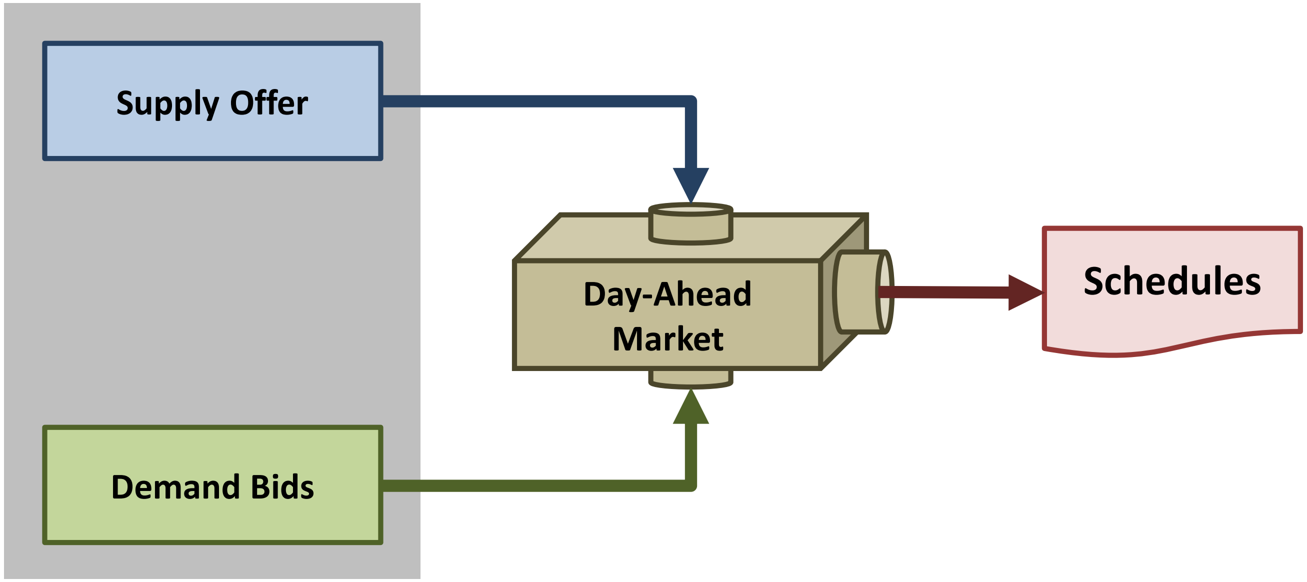
- ***Objective***: Present an alternative approach to devise bidding strategies in day-ahead electricity markets.

- ***Challenge***: Characterize the uncertainty on Rival competitors' bidding.
  - Assume perfect information (deterministic approach) may lead to meaningless solutions;
  - Construct a probability distribution to characterize strategic behavior is a very challenging task;

- ***Contributions***:
  1. Present a novel risk-averse model based on robust optimization under polyhedral uncertainty set for devising optimal bidding strategies in sealed-bid uniform-price auctions with multiple divisible products
  2. Provide a single-level equivalent formulation suitable for decomposition techniques suitable for based on available commercial solvers.
  3. Develop an efficient solution methodology based on column-and-constraint generation (CCG) algorithm.

Day-Ahead Electricity Markets – General Scheme

- The general idea is simple:
  - Participants submit buy (demand) and sell (producers) orders.
  - The market is cleared by a central (system/market) operator.
  - Denition of the uniform clearing price and amount due to each participant.



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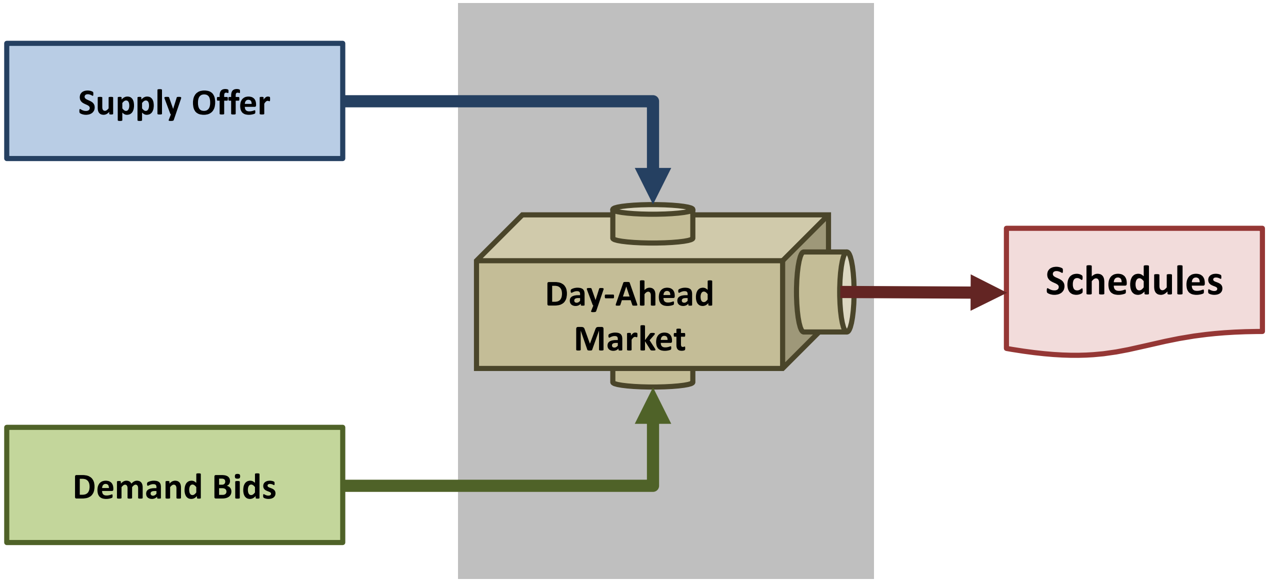

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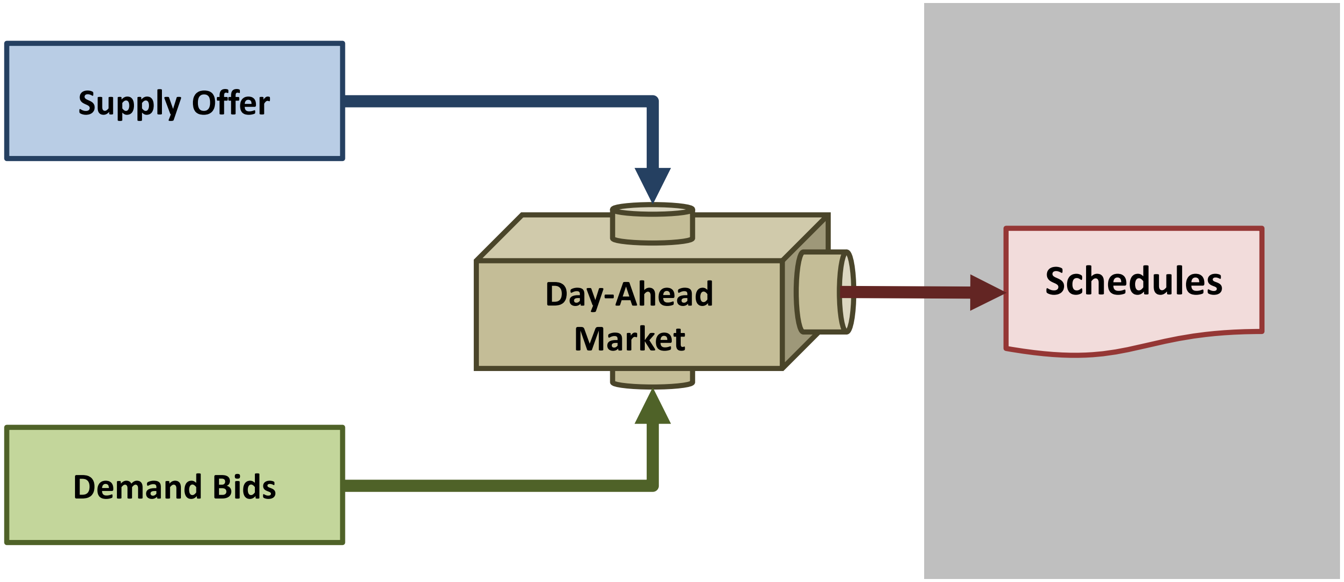
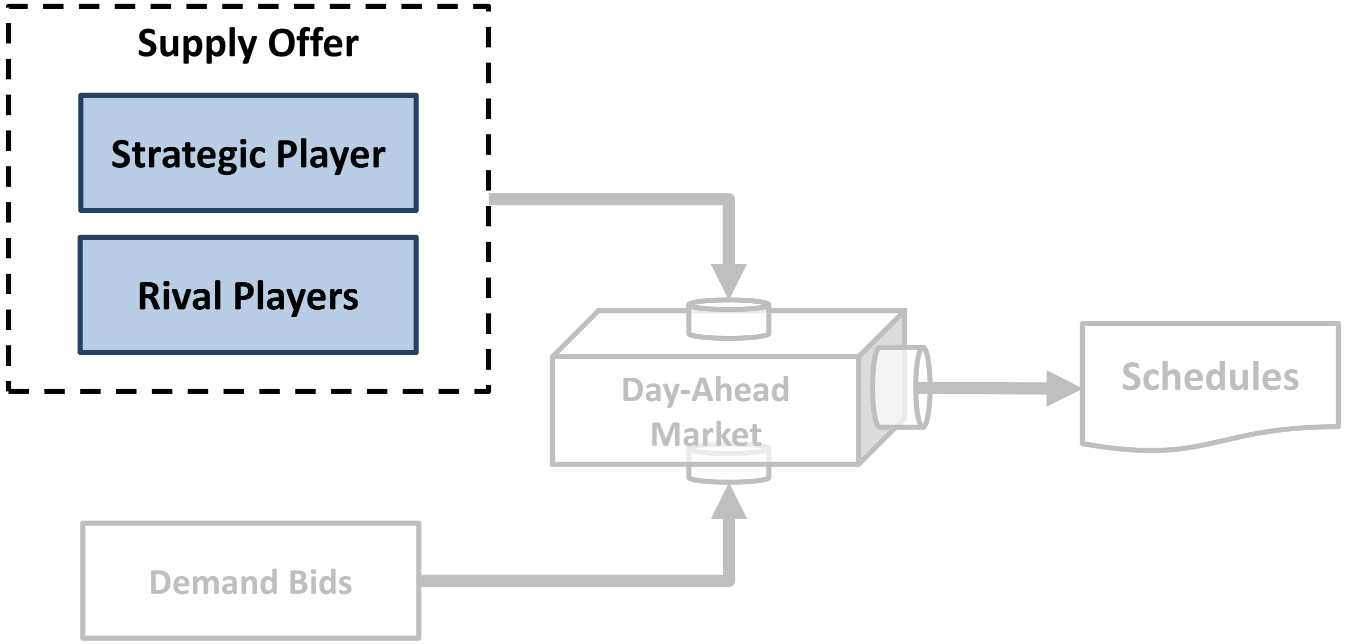


Day-Ahead Electricity Markets – General Scheme

- **Our goal**: develop a new methodology to devise a profit-maximizing strategic offer for a subset of supply companies, hereinafter called *Strategic Player*.
  - The remainders will be called *Rival Players*.



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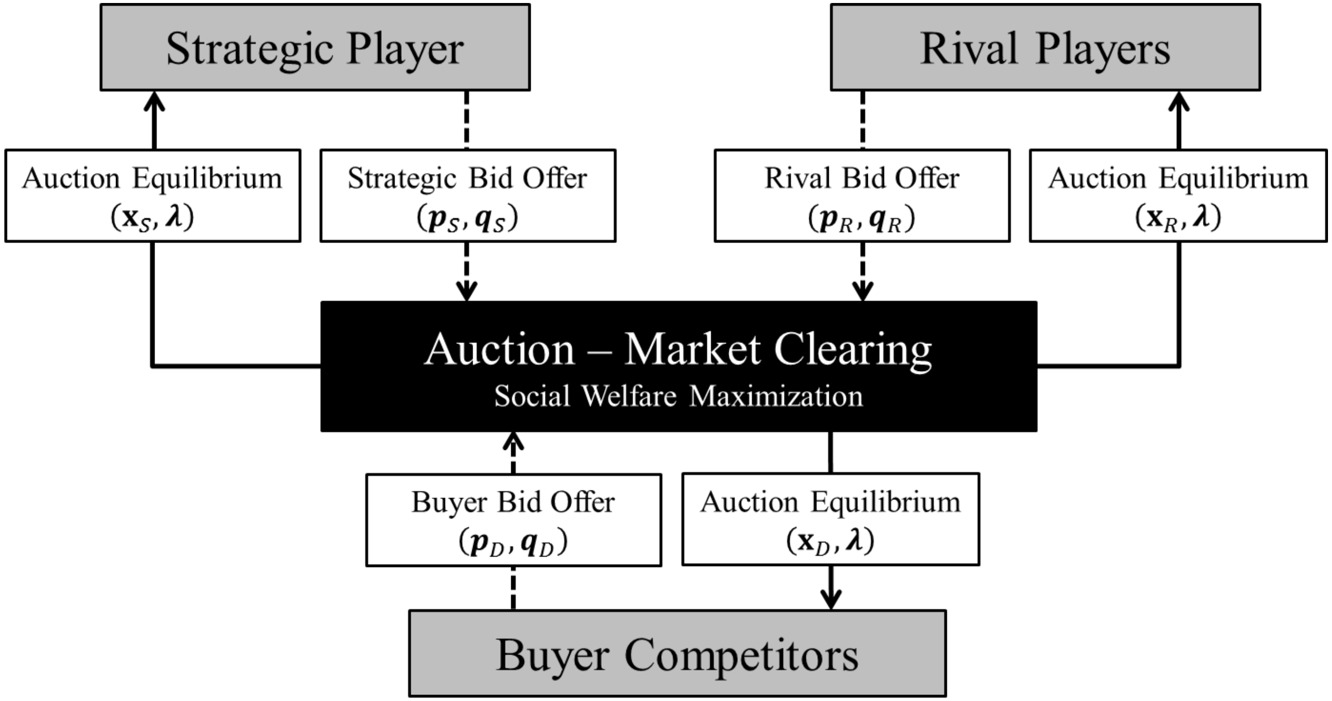

Auction Problem – Linear and Continuous

- The mathematical formulation of the day-ahead market assumes a single node network with (an a priori known) inelastic demand in a single-period setting.

$$\min_{\boldsymbol{g}^S, \boldsymbol{g}^R} \sum_{j \in \mathcal{N}_S} p_j^S g_j^S + \sum_{i \in \mathcal{N}_R} p_i^R g_i^R$$

subject to:

$$\sum_{j \in \mathcal{N}_S} g_j^S + \sum_{i \in \mathcal{N}_R} g_i^R = d; \qquad : \lambda$$

$$0 \le g_j^S \le q_j^S, \qquad : \left(\overline{\lambda}_j^S, \underline{\lambda}_j^S\right) \qquad \forall\, j \in \mathcal{N}_S;$$

$$0 \le g_i^R \le q_i^R, \qquad : \left(\overline{\lambda}_i^R, \underline{\lambda}_i^R\right) \qquad \forall\, i \in \mathcal{N}_R.$$

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Prices submitted to the auction – social welfare maximization

- $p_j^S \rightarrow$ Strategic player $(j \in \mathcal{N}_S)$ price offer;
- $p_i^R \rightarrow$ Rival players $(i \in \mathcal{N}_R)$ price offer;
- $p_D \rightarrow$ Buyer (demand) price bid

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Quantity submitted to the auction – constrains the amount sold/bought for each player

- q_j^S → Strategic player ($j \in \mathcal{N}_S$) quantity offer;
- q_i^R → Rival players ($i \in \mathcal{N}_R$) quantity offer;
- $\boldsymbol{q_D}$ → Buyer (demand) quantity bid

Auction Problem – Linear and Continuous

- The foundation of the business environment is a competitive market endowed with a sealed-bid uniform-price auction of multiple divisible goods formulated as a linear programming problem.

$$\min_{g^S, g^R} \sum_{j \in \mathcal{N}_S} p_j^S g_j^S + \sum_{i \in \mathcal{N}_R} p_i^R g_i^R$$

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Optimal Economic Dispatch – auction solution

- g_j^S → Strategic player ($j \in \mathcal{N}_S$) dispatch;
- g_i^R → Rival players ($i \in \mathcal{N}_R$) dispatch;
- d → Buyer (demand) “responsive” consumption;

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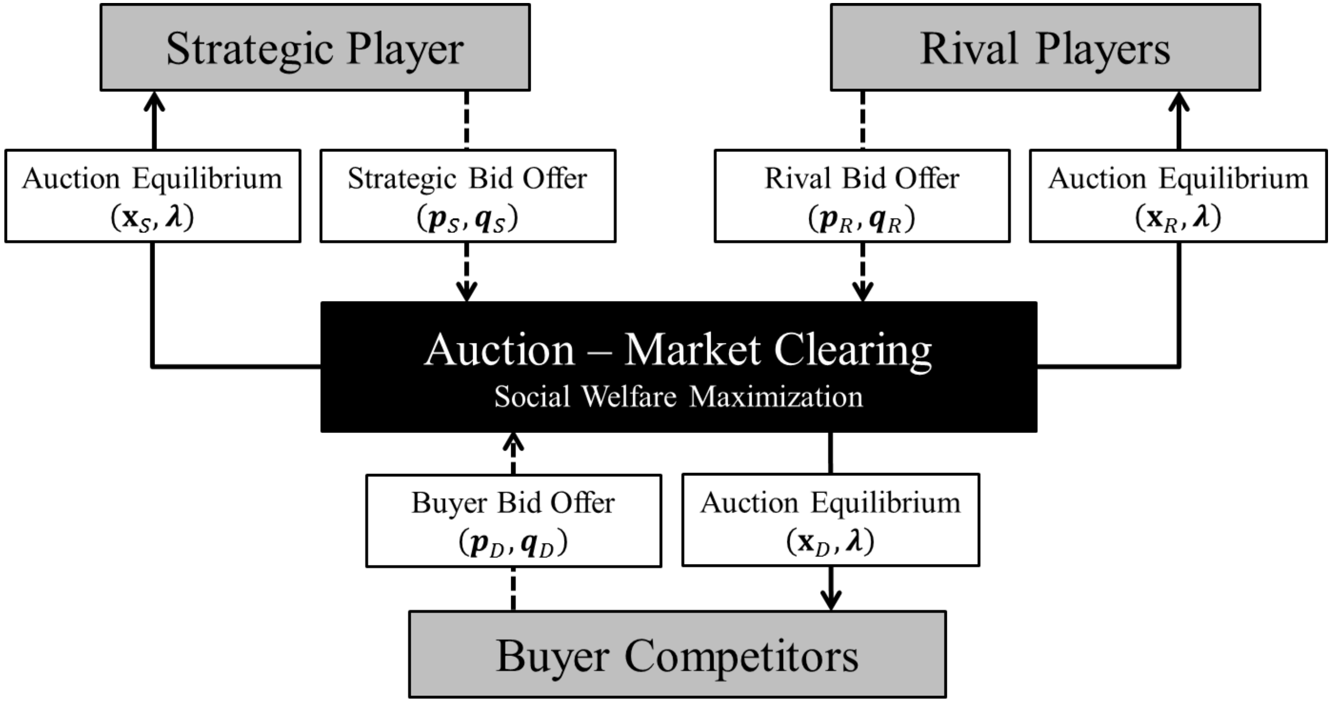
λ: Clearing energy price – uniform price

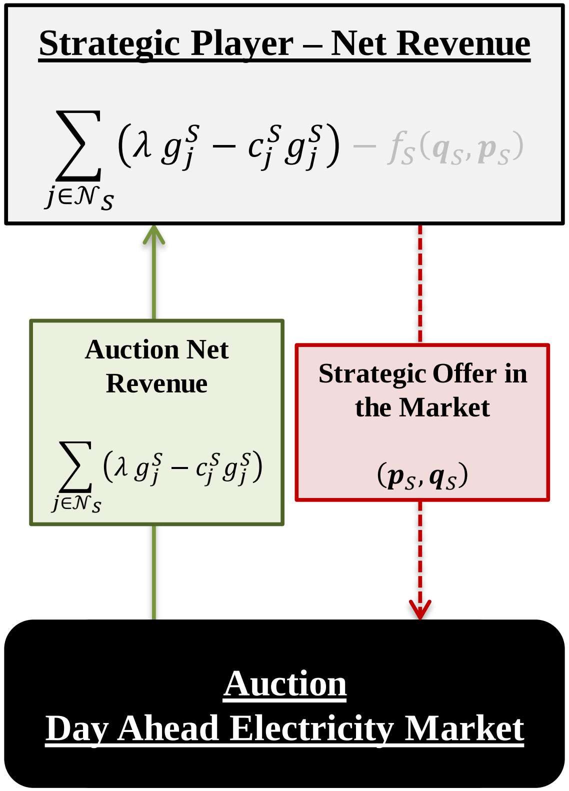
Profit-Maximizing Strategic Offer Problem

- <u>Major challenge</u>: What is the optimal offer that maximizes the *Strategic Player* profit?



Strategic Player Net Revenue – Auction

- The Strategic Player net revenue is composed by three terms:
  1. Day-ahead market revenue of strategic player (unit) $j \in \mathcal{N}_S$: $\lambda\, g_j^S$
  2. Production (linear) costs of strategic player (unit) $j \in \mathcal{N}_S$: $c_j^S\, g_j^S$
  3. Bid costs: $f_S(\boldsymbol{q}_S, \boldsymbol{p}_S)$



Bidding Problem – Scheme

- <u>Major challenge: how to bid into the market in order to maximize the strategic player profit?</u>

- Assuming a feasible bid region $\mathcal{O}_S$, the optimal bidding problem resumes to:

$$\max_{\boldsymbol{p}^S,\boldsymbol{q}^S} \sum_{j\in\mathcal{N}_S} \left(\lambda\, g_j^S - c_j^S g_j^S\right)$$

subject to:

$$(\boldsymbol{p}^S,\boldsymbol{q}^S) \in \mathcal{O}_S$$

Strategic Bid: $(\boldsymbol{p}^S,\boldsymbol{q}^S)$

$$\min_{\boldsymbol{g}^S,\boldsymbol{g}^R} \sum_{j\in\mathcal{N}_S} p_j^S g_j^S + \sum_{i\in\mathcal{N}_R} p_i^R g_i^R$$

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Auction Solution: $(\boldsymbol{g}^S,\lambda)$

Profit-Maximizing Strategic Offer Problem

- Major challenge: how to bid into the market in order to maximize the strategic player profit?

- Let $\mathcal{M}(\boldsymbol{p}^S, \boldsymbol{q}^S, \boldsymbol{p}^R, \boldsymbol{q}^R)$ be a (non-empty) set of all optimal points $\boldsymbol{g}^S$ and respective dual variables λ of the auction problem.

$$\mathcal{M}(\boldsymbol{p}^S, \boldsymbol{q}^S, \boldsymbol{p}^R, \boldsymbol{q}^R) = \{(\boldsymbol{g}^S, \lambda) \mid (\boldsymbol{g}^S, \lambda) \text{ solves the day—ahead problem}\}$$

$$\mathcal{M}(\boldsymbol{p}^S, \boldsymbol{q}^S, \boldsymbol{p}^R, \boldsymbol{q}^R) = \arg\min_{\boldsymbol{g}^S, \boldsymbol{g}^R} \sum_{j \in \mathcal{N}_S} p_j^S g_j^S + \sum_{i \in \mathcal{N}_R} p_i^R g_i^R$$

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Bidding Problem with Auction Solution

- <u>Main question</u>: how to optimally bid into the market in order to maximize the strategic player profit?

- Let $\mathcal{M}(\boldsymbol{p}_S, \boldsymbol{q}_S, \boldsymbol{p}_R, \boldsymbol{q}_R)$ be a (non-empty) set of all optimal points $\mathbf{x}_S$ and respective dual variables $\boldsymbol{\lambda}$ of the auction problem.

$$\mathcal{M}(\boldsymbol{p}_S, \boldsymbol{q}_S, \boldsymbol{p}_R, \boldsymbol{q}_R) = \{(\mathbf{x}_S, \boldsymbol{\lambda}) \in \mathbb{R}^{N_S} \times \mathbb{R}^M \mid (\mathbf{x}_S, \boldsymbol{\lambda}) \text{ solves the auction problem}\}$$

- We can write the bidding problem as follows:

$$\max_{\boldsymbol{p}^S, \boldsymbol{q}^S, \boldsymbol{g}^S, \lambda} \sum_{j \in \mathcal{N}_S} \left(\lambda\, g_j^S - c_j^S g_j^S\right)$$

subject to:

$$(\boldsymbol{p}^S, \boldsymbol{q}^S) \in \mathcal{O}_S$$

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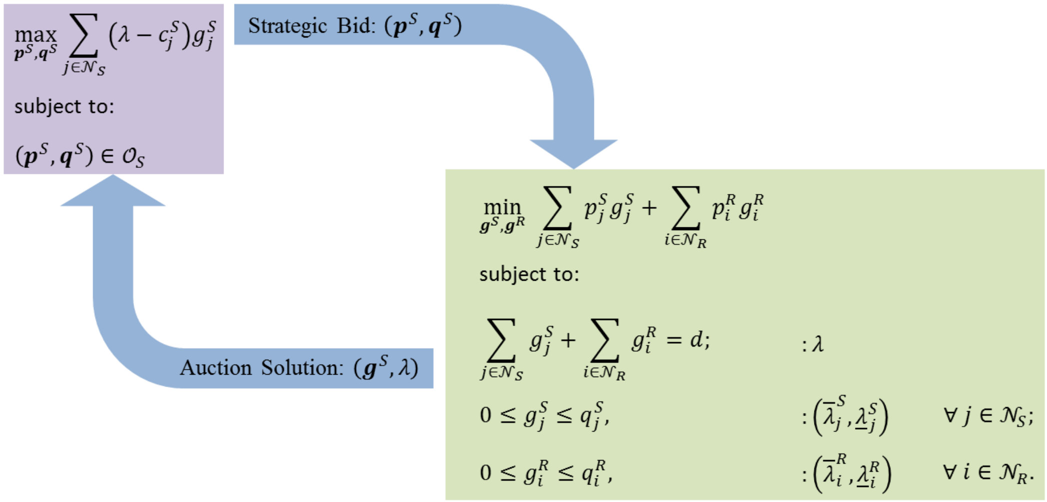

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How to represent the uncertainty on Rivals' bidding variables $(\boldsymbol{p}_R, \boldsymbol{q}_R)$?

Several works assume:

- perfect information (deterministic approach) or;
- available a probability distribution (stochastic approach).

Proposed Model – Robust Approach

- Key challenge: how to represent the uncertainty on rivals bidding variables $(\boldsymbol{p}_R, \boldsymbol{q}_R)$?

- Let $(\tilde{\boldsymbol{p}}_R, \tilde{\boldsymbol{q}}_R)$ be the vector of uncertain rival price/quantity bids.

- The optimal bidding problem under uncertainty on rival's bid can be formulated as:

$$\max_{\boldsymbol{p}_S, \boldsymbol{q}_S, \tilde{\mathbf{x}}_S, \tilde{\boldsymbol{\lambda}}} \Phi\left(\sum_{j \in \mathcal{N}_S} (\lambda\, g_j^S - c_j^S g_j^S)\right)$$

subject to:

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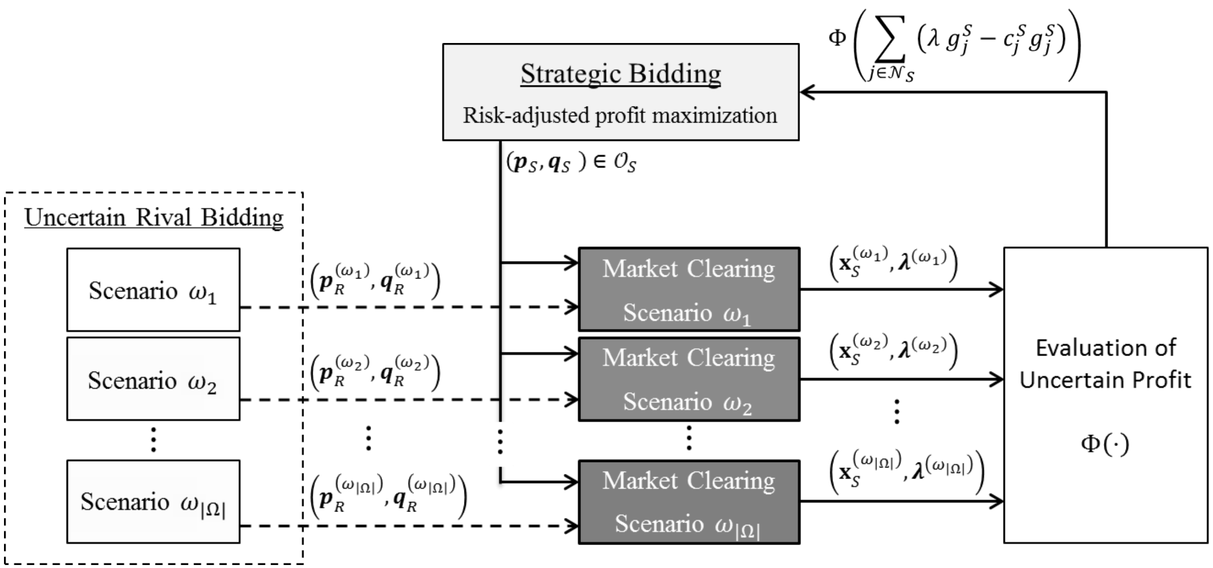

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subject to:

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$$(\mathbf{x}_S, \boldsymbol{\lambda}) \in \mathcal{M}(\boldsymbol{p}_S, \boldsymbol{q}_S, \tilde{\boldsymbol{p}}_R, \tilde{\boldsymbol{q}}_R)$$

We argue that characterizing an adequate probability distribution for the rival bids $(\tilde{\boldsymbol{p}}_R, \tilde{\boldsymbol{q}}_R)$ is a hard task due to its complexity.

Proposed Model – Robust Approach

- In this context *Robust Optimization* emerges as a powerful tool.
- Let $\mathcal{O}_R$ be set the of feasible ("credible") bids of *Rival Players*. Then, the proposed model is:

$$\max_{\boldsymbol{p}^S,\boldsymbol{q}^S} \left\{ \min_{\boldsymbol{p}^R,\boldsymbol{q}^R,\boldsymbol{g}^S,\lambda} \sum_{j\in\mathcal{N}_S} \left(\lambda\, g_j^S - c_j^S g_j^S\right) \right.$$

subject to:

$$(\boldsymbol{p}^R, \boldsymbol{q}^R) \in \mathcal{O}_R$$

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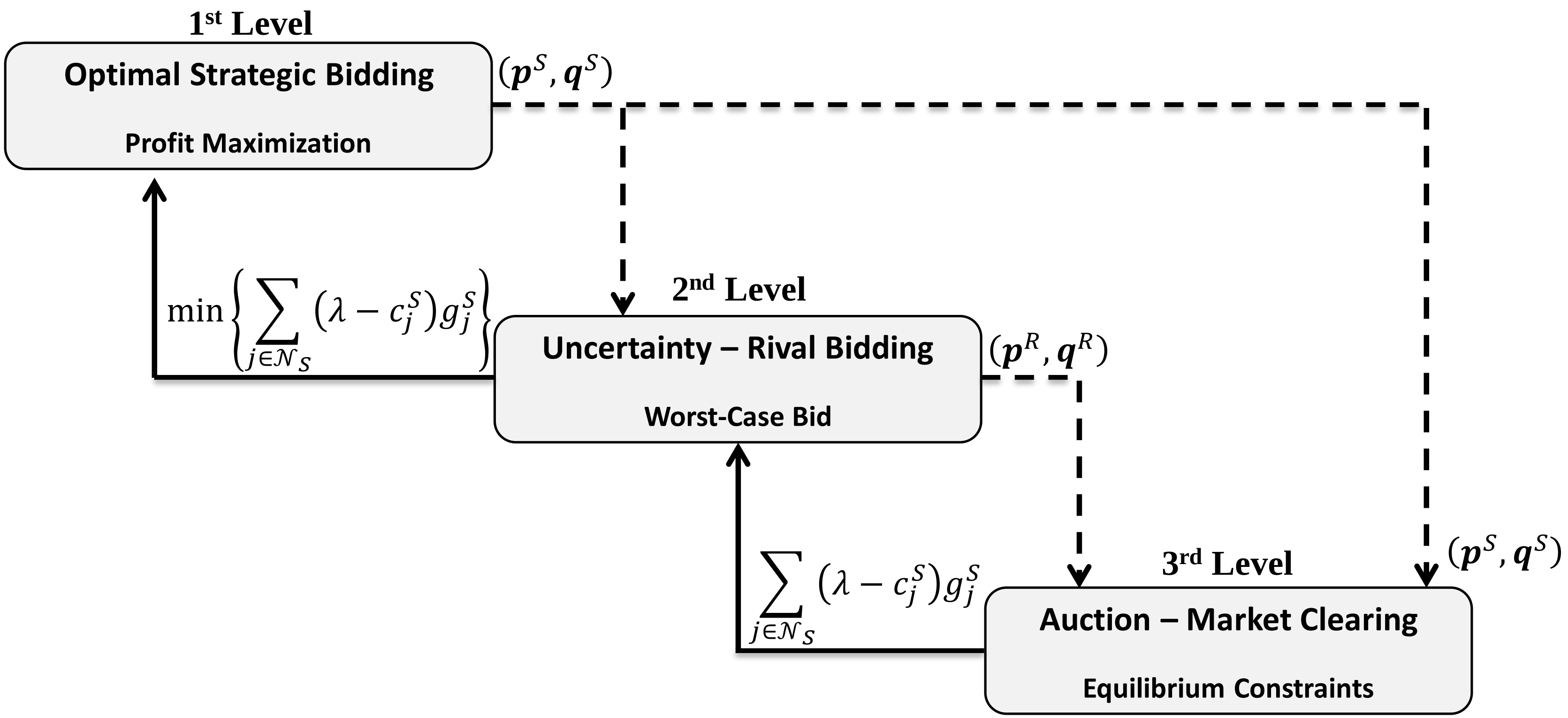
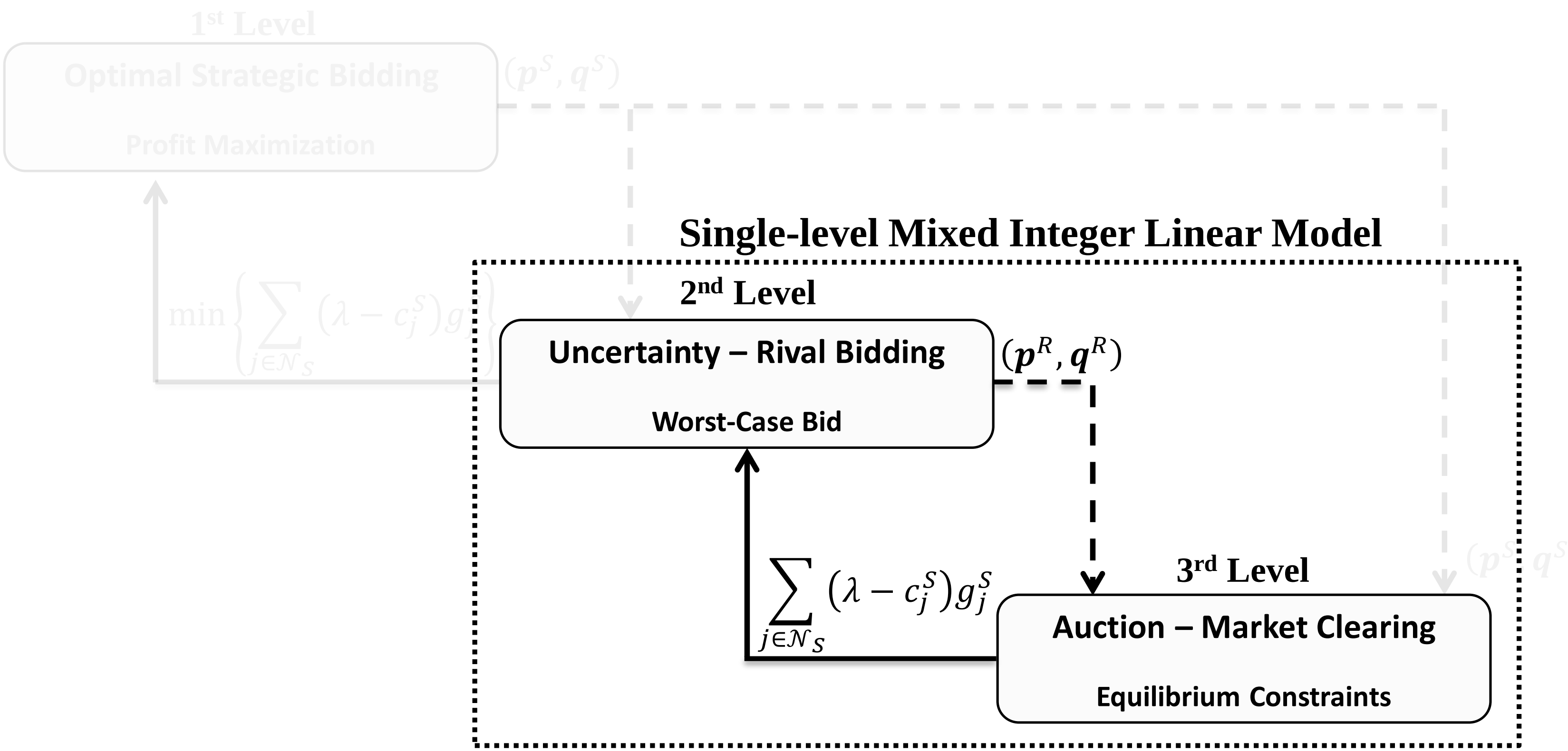


Proposed Model – Robust Approach

- How to solve the proposed model?
  - Transform the second and third-level problems into a single-level mixed integer linear model.



Solution Methodology – KKT System

- How to solve the proposed model?

$$\max_{\boldsymbol{p}^S,\boldsymbol{q}^S} \left\{ \min_{\boldsymbol{p}^R,\boldsymbol{q}^R,\boldsymbol{g}^S,\lambda} \sum_{j\in\mathcal{N}_S} \left(\lambda\, g_j^S - c_j^S g_j^S\right) \right.$$

subject to:

$$(\boldsymbol{p}^R,\boldsymbol{q}^R) \in \mathcal{O}_R$$

$$\left.(\boldsymbol{g}^S,\lambda) \in \mathcal{M}(\boldsymbol{p}^S,\boldsymbol{q}^S,\boldsymbol{p}^R,\boldsymbol{q}^R) \right\}$$

subject to:

$$(\boldsymbol{p}^S,\boldsymbol{q}^S) \in \mathcal{O}_S$$

Firstly, recall that the day-ahead problem is linear and continuous.

We can replace $\mathcal{M}(\boldsymbol{p}^S,\boldsymbol{q}^S,\boldsymbol{p}^R,\boldsymbol{q}^R)$ by its KKT optimality conditions

<u>Day-Ahead Problem</u>

$$\min_{\boldsymbol{g}^S,\boldsymbol{g}^R} \sum_{j\in\mathcal{N}_S} p_j^S g_j^S + \sum_{i\in\mathcal{N}_R} p_i^R g_i^R$$

subject to:

$$\sum_{j\in\mathcal{N}_S} g_j^S + \sum_{i\in\mathcal{N}_R} g_i^R = d; \qquad :\lambda$$

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Solution Methodology – KKT System

- How to solve the proposed model?

Primal Constraints	Stationarity Constraints
	Dual Constraints
Complementarity Constraints	

$$\max_{\boldsymbol{p}^S,\boldsymbol{q}^S} \left\{ \min_{\substack{\boldsymbol{p}^R,\boldsymbol{q}^R,\boldsymbol{g}^S,\lambda \\ \overline{\boldsymbol{\lambda}}^S,\underline{\boldsymbol{\lambda}}^S,\overline{\boldsymbol{\lambda}}^R,\underline{\boldsymbol{\lambda}}^R,\lambda}} \sum_{j\in\mathcal{N}_S} (\lambda\, g_j^S - c_j^S g_j^S) \right.$$

subject to:

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$$p_j^S - \lambda + \overline{\lambda}_j^S - \underline{\lambda}_j^S = 0 \quad \forall\, j \in \mathcal{N}_S;$$

$$p_i^R - \lambda + \overline{\lambda}_i^R - \underline{\lambda}_i^R = 0, \quad \forall\, i \in \mathcal{N}_R;$$

$$\overline{\lambda}_j^S, \underline{\lambda}_j^S, \underline{\lambda}_i^R, \overline{\lambda}_i^R \ge 0 \quad \forall\, j \in \mathcal{N}_S, i \in \mathcal{N}_R;$$

$$(q_j^S - g_j^S)\overline{\lambda}_j^S = 0, \quad \forall\, j \in \mathcal{N}_S; \qquad (q_i^R - g_i^R)\overline{\lambda}_i^R = 0, \quad \forall\, i \in \mathcal{N}_R;$$

$$\left. g_j^S \underline{\lambda}_j^S = 0 \quad \forall\, j \in \mathcal{N}_S; \qquad g_i^R \underline{\lambda}_i^R = 0 \quad \forall\, i \in \mathcal{N}_R; \right\}$$

subject to:

$$(\boldsymbol{p}^S,\boldsymbol{q}^S) \in \mathcal{O}_S$$

Bilinear Product – Day-Ahead Revenue

- How to solve the proposed model?

$$\max_{\boldsymbol{p}^S,\boldsymbol{q}^S} \left\{ \min_{\substack{\boldsymbol{p}^R,\boldsymbol{q}^R,\boldsymbol{g}^S,\lambda \\ \overline{\lambda}^S,\underline{\lambda}^S,\overline{\lambda}^R,\underline{\lambda}^R,\lambda}} \sum_{j\in\mathcal{N}_S} \left(\lambda\, g_j^S - c_j^S g_j^S\right) \right.$$

subject to:

$$(\boldsymbol{p}^R,\boldsymbol{q}^R) \in \mathcal{O}_R$$

$$\sum_{j\in\mathcal{N}_S} g_j^S + \sum_{i\in\mathcal{N}_R} g_i^R = d; \qquad p_j^S - \lambda + \overline{\lambda}_j^S - \underline{\lambda}_j^S = 0 \qquad \forall\, j \in \mathcal{N}_S;$$

$$0 \le g_j^S \le q_j^S, \qquad \forall\, j \in \mathcal{N}_S; \qquad p_i^R - \lambda + \overline{\lambda}_i^R - \underline{\lambda}_i^R = 0, \qquad \forall\, i \in \mathcal{N}_R;$$

$$0 \le g_i^R \le q_i^R, \qquad \forall\, i \in \mathcal{N}_R; \qquad \overline{\lambda}_j^S, \underline{\lambda}_j^S, \underline{\lambda}_i^R, \overline{\lambda}_i^R \ge 0 \qquad \forall\, j \in \mathcal{N}_S, i \in \mathcal{N}_R;$$

$$(q_j^S - g_j^S)\bar{\lambda}_j^S = 0, \qquad \forall\, j \in \mathcal{N}_S; \qquad (q_i^R - g_i^R)\bar{\lambda}_i^R = 0, \qquad \forall\, i \in \mathcal{N}_R;$$

$$\left. g_j^S \underline{\lambda}_j^S = 0 \qquad \forall\, j \in \mathcal{N}_S; \qquad g_i^R \underline{\lambda}_i^R = 0 \qquad \forall\, i \in \mathcal{N}_R; \right\}$$

subject to:

$$(\boldsymbol{p}^S,\boldsymbol{q}^S) \in \mathcal{O}_S$$

Bilinear Product – Day-Ahead Revenue

- How to solve the proposed model?

$$\max_{p^S,q^S} \left\{ \min_{\substack{p^R,q^R,g^S,\lambda \\ \bar{\lambda}^S,\underline{\lambda}^S,\bar{\lambda}^R,\underline{\lambda}^R,\lambda}} \sum_{j\in\mathcal{N}_S} \left(p_j^S g_j^S + \bar{\lambda}_j^S q_j^S - c_j^S g_j^S \right) \right.$$

subject to:

$$(\boldsymbol{p}^R,\boldsymbol{q}^R) \in \mathcal{O}_R$$

$$\sum_{j\in\mathcal{N}_S} g_j^S + \sum_{i\in\mathcal{N}_R} g_i^R = d;$$

$$p_j^S - \lambda + \bar{\lambda}_j^S - \underline{\lambda}_j^S = 0 \qquad \forall j \in \mathcal{N}_S;$$

$$0 \le g_j^S \le q_j^S, \qquad \forall j \in \mathcal{N}_S; \qquad p_i^R - \lambda + \bar{\lambda}_i^R - \underline{\lambda}_i^R = 0, \qquad \forall i \in \mathcal{N}_R;$$

$$0 \le g_i^R \le q_i^R, \qquad \forall i \in \mathcal{N}_R; \qquad \bar{\lambda}_j^S, \underline{\lambda}_j^S, \underline{\lambda}_i^R, \bar{\lambda}_i^R \ge 0 \qquad \forall j \in \mathcal{N}_S, i \in \mathcal{N}_R;$$

$$(q_j^S - g_j^S)\bar{\lambda}_j^S = 0, \qquad \forall j \in \mathcal{N}_S; \qquad (q_i^R - g_i^R)\bar{\lambda}_i^R = 0, \qquad \forall i \in \mathcal{N}_R;$$

$$\left. g_j^S \underline{\lambda}_j^S = 0 \qquad \forall j \in \mathcal{N}_S; \qquad g_i^R \underline{\lambda}_i^R = 0 \qquad \forall i \in \mathcal{N}_R; \right\}$$

subject to:

$$(\boldsymbol{p}^S,\boldsymbol{q}^S) \in \mathcal{O}_S$$

Bilinear product: $\lambda\, g_j^S$

We can combine these equations and write the bilinear product as:

$$\lambda\, g_j^S = p_j^S g_j^S + \bar{\lambda}_j^S q_j^S$$

Complementarity Constraints

- How to solve the proposed model?

$$\max_{\boldsymbol{p}^S,\boldsymbol{q}^S} \left\{ \min_{\substack{\boldsymbol{p}^R,\boldsymbol{q}^R,\boldsymbol{g}^S,\lambda \\ \overline{\lambda}^S,\underline{\lambda}^S,\overline{\lambda}^R,\underline{\lambda}^R,\lambda}} \sum_{j\in\mathcal{N}_S} \left(p_j^S g_j^S + \overline{\lambda}_j^S q_j^S - c_j^S g_j^S\right) \right.$$

subject to:

$$(\boldsymbol{p}^R, \boldsymbol{q}^R) \in \mathcal{O}_R$$

$$\sum_{j\in\mathcal{N}_S} g_j^S + \sum_{i\in\mathcal{N}_R} g_i^R = d; \qquad p_j^S - \lambda + \overline{\lambda}_j^S - \underline{\lambda}_j^S = 0 \qquad \forall\, j \in \mathcal{N}_S;$$

$$0 \le g_j^S \le q_j^S, \quad \forall\, j \in \mathcal{N}_S; \qquad p_i^R - \lambda + \overline{\lambda}_i^R - \underline{\lambda}_i^R = 0, \quad \forall\, i \in \mathcal{N}_R;$$

$$0 \le g_i^R \le q_i^R, \quad \forall\, i \in \mathcal{N}_R; \qquad \overline{\lambda}_j^S, \underline{\lambda}_j^S, \underline{\lambda}_i^R, \overline{\lambda}_i^R \ge 0 \quad \forall\, j \in \mathcal{N}_S, i \in \mathcal{N}_R;$$

$$(q_j^S - g_j^S)\overline{\lambda}_j^S = 0, \quad \forall\, j \in \mathcal{N}_S; \qquad (q_i^R - g_i^R)\overline{\lambda}_i^R = 0, \quad \forall\, i \in \mathcal{N}_R;$$

$$\left. g_j^S \underline{\lambda}_j^S = 0 \quad \forall\, j \in \mathcal{N}_S; \qquad g_i^R \underline{\lambda}_i^R = 0 \quad \forall\, i \in \mathcal{N}_R; \right\}$$

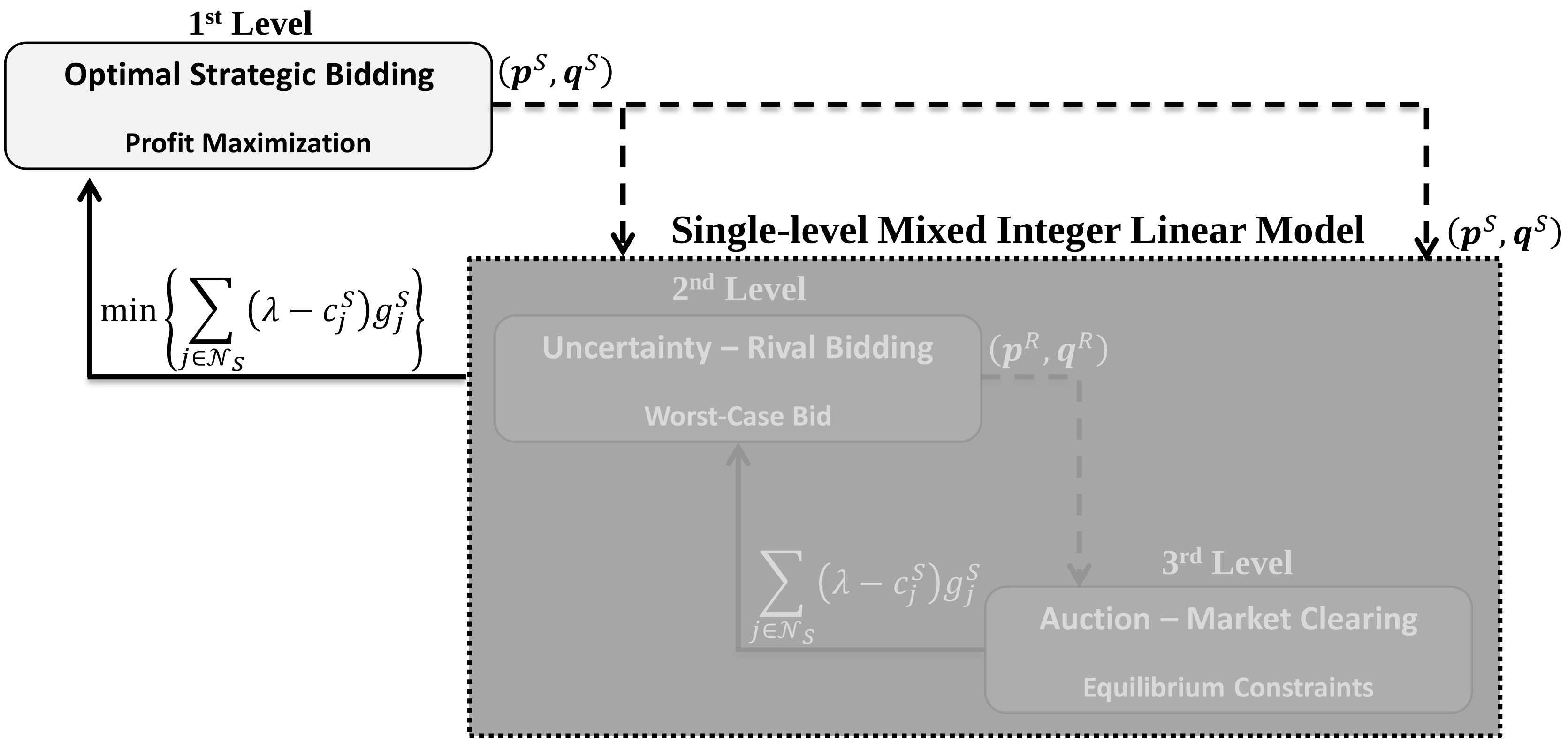
subject to:

$$(\boldsymbol{p}^S, \boldsymbol{q}^S) \in \mathcal{O}_S$$

The complementarity constraints can be (exact) linearized using McCormick envelopes and binary variables.

Proposed Model – Robust Approach

- How to solve the proposed model?
  - Apply Column-and-Constraint Generation Algorithms.



Column-and-Constraint Generation Algorithm

- To ease the presentation of the Column-and-Constraint Generation Algorithm, the two-level model will be presented in a compact way.

$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\substack{\boldsymbol{z}_L \geq \boldsymbol{0} \\ \boldsymbol{u} \in \mathbb{B}}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_U^T \boldsymbol{B} \boldsymbol{z}_L \,\middle|\, \boldsymbol{L} \boldsymbol{z}_L \geq \boldsymbol{E} \boldsymbol{z}_U + \boldsymbol{F} \boldsymbol{u} + \boldsymbol{b} \right\} \right\}$$

- Where $\mathbb{B}$ is a binary set.

Column-and-Constraint Generation Algorithm

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$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\substack{\boldsymbol{z}_L \geq \boldsymbol{0} \\ \boldsymbol{u} \in \mathbb{B}}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_U^T \boldsymbol{B} \boldsymbol{z}_L \;\middle|\; \boldsymbol{L}\boldsymbol{z}_L \geq \boldsymbol{E}\boldsymbol{z}_U + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{b} \right\} \right\}$$

- Where $\mathbb{B}$ is a binary set.
- Enquivalently, we can "split" the second-level problem into two problems:

$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\boldsymbol{u} \in \mathbb{B}} \left\{ \min_{\boldsymbol{z}_L \geq \boldsymbol{0}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_U^T \boldsymbol{B} \boldsymbol{z}_L \;\middle|\; \boldsymbol{L}\boldsymbol{z}_L \geq \boldsymbol{E}\boldsymbol{z}_U + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{b} : \boldsymbol{\theta} \right\} \right\} \right\}$$

- With $\boldsymbol{\theta}$ the dual variable of the inner-problem constraints.

Column-and-Constraint Generation Algorithm

- To ease the presentation of the Column-and-Constraint Generation Algorithm, the two-level model will be presented in a compact way.

$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\substack{\boldsymbol{z}_L \geq \boldsymbol{0} \\ \boldsymbol{u} \in \mathbb{B}}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_U^T \boldsymbol{B} \boldsymbol{z}_L \,\middle|\, \boldsymbol{L} \boldsymbol{z}_L \geq \boldsymbol{E} \boldsymbol{z}_U + \boldsymbol{F} \boldsymbol{u} + \boldsymbol{b} \right\} \right\}$$

- Where $\mathbb{B}$ is a binary set.
- Enquivalently, we can "split" the second-level problem into two problems:

$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\boldsymbol{u} \in \mathbb{B}} \left\{ \min_{\boldsymbol{z}_L \geq \boldsymbol{0}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_U^T \boldsymbol{B} \boldsymbol{z}_L \,\middle|\, \boldsymbol{L} \boldsymbol{z}_L \geq \boldsymbol{E} \boldsymbol{z}_U + \boldsymbol{F} \boldsymbol{u} + \boldsymbol{b} : \boldsymbol{\theta} \right\} \right\} \right\}$$

- With $\boldsymbol{\theta}$ the dual variable of the inner-problem constraints.
- By duality theory,

$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\boldsymbol{u} \in \mathbb{B}} \left\{ \max_{\boldsymbol{\theta} \geq \boldsymbol{0}} \left\{ \boldsymbol{\theta}^T (\boldsymbol{E} \boldsymbol{z}_U + \boldsymbol{F} \boldsymbol{u} + \boldsymbol{b}) \mid \boldsymbol{L}^T \boldsymbol{\theta} \leq \boldsymbol{g} + \boldsymbol{B}^T \boldsymbol{z}_U \right\} \right\} \right\}$$

Column-and-Constraint Generation Algorithm

- To ease the presentation of the Column-and-Constraint Generation Algorithm, the two-level model will be presented in a compact way.

$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\substack{\boldsymbol{z}_L \geq \boldsymbol{0} \\ \boldsymbol{u} \in \mathbb{B}}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_U^T \boldsymbol{B} \boldsymbol{z}_L \,\middle|\, \boldsymbol{L}\boldsymbol{z}_L \geq \boldsymbol{E}\boldsymbol{z}_U + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{b} \right\} \right\}$$

- Where $\mathbb{B}$ is a binary set.
- Enquivalently, we can “split” the second-level problem into two problems:

$$\max_{\boldsymbol{z}_U \in \mathcal{O}_S} \left\{ \min_{\boldsymbol{u} \in \mathbb{B}} \left\{ \min_{\boldsymbol{z}_L \geq \boldsymbol{0}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_U^T \boldsymbol{B} \boldsymbol{z}_L \,\middle|\, \boldsymbol{L}\boldsymbol{z}_L \geq \boldsymbol{E}\boldsymbol{z}_U + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{b} : \boldsymbol{\theta} \right\} \right\} \right\}$$

- With $\boldsymbol{\theta}$ the dual variable of the inner-problem constraints.
- Equivalently, middle and inner problems on the constraints

$$\max_{\boldsymbol{z}_U, \eta} \eta$$

subject to:

$$\eta \leq \min_{\boldsymbol{u} \in \mathbb{B}} \left\{ \max_{\boldsymbol{\theta} \geq \boldsymbol{0}} \left\{ \boldsymbol{\theta}^T (\boldsymbol{E}\boldsymbol{z}_U + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{b}) \mid \boldsymbol{L}^T \boldsymbol{\theta} \leq \boldsymbol{g} + \boldsymbol{B}^T \boldsymbol{z}_U \right\} \right\}$$

$$\boldsymbol{z}_U \in \mathcal{O}_S$$

Column-and-Constraint Generation Algorithm

- To ease the presentation of the Column-and-Constraint Generation Algorithm, the two-level model will be presented in a compact way.

$$\max_{z_U \in \mathcal{O}_S} \left\{ \min_{\substack{z_L \geq 0 \\ u \in \mathbb{B}}} \{ g^T z_L + z_U^T B z_L \mid L z_L \geq E z_U + F u + b \} \right\}$$

- Where $\mathbb{B}$ is a binary set.
- Enquivalently, we can "split" the second-level problem into two problems:

$$\max_{z_U \in \mathcal{O}_S} \left\{ \min_{u \in \mathbb{B}} \left\{ \min_{z_L \geq 0} \{ g^T z_L + z_U^T B z_L \mid L z_L \geq E z_U + F u + b : \theta \} \right\} \right\}$$

- With θ the dual variable of the inner-problem constraints.
- Since $\mathbb{B}$ is a binary set, we can rewrite in an equivalent form with an exponential set of constraints

$$\max_{z_U, \eta} \eta$$

subject to:

$$\eta \leq \max_{\theta_u \geq 0} \{ \theta_u^T (E z_U + F u + b) \mid L^T \theta_u \leq g + B^T z_U \} \qquad \forall\, u \in \mathbb{B}$$

$$z_U \in \mathcal{O}_S$$

CCG Algorithm – Full Problem

- Therefore, the following single-level formulation can be written

$$\varphi^* = \max_{\boldsymbol{z}_U, \eta, \boldsymbol{\theta}_{\boldsymbol{u}}} \; -f_S(\boldsymbol{z}_U) + \eta$$

subject to:

$$\eta \leq \boldsymbol{\theta}_{\boldsymbol{u}}^T(\boldsymbol{E}\boldsymbol{z}_U + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{b}) \qquad \forall\, \boldsymbol{u} \in \mathbb{B}$$

$$\boldsymbol{L}^T\boldsymbol{\theta}_{\boldsymbol{u}} \leq \boldsymbol{g} + \boldsymbol{B}^T\boldsymbol{z}_U \qquad \forall\, \boldsymbol{u} \in \mathbb{B}$$

$$\boldsymbol{\theta}_{\boldsymbol{u}} \geq \boldsymbol{0} \qquad \forall\, \boldsymbol{u} \in \mathbb{B}$$

$$\boldsymbol{z}_U \in \mathcal{O}_S$$

- The resulting model is a large-scale optimization proble with (pottentially) exponential set of constraints, but suitable for Column-and-Constraint Generation Algorithm.

CCG Algorithm – Master Problem

- Then, consider $\mathbb{B}_k \subset \mathbb{B}$ and the following optimization problem:

$$\overline{\varphi}_k = \max_{\boldsymbol{z}_U, \eta, \boldsymbol{\theta}_{\boldsymbol{u}}} \; -f_S(\boldsymbol{z}_U) + \eta$$

subject to:

$$\eta \leq \boldsymbol{\theta}_{\boldsymbol{u}}^T(\boldsymbol{E}\boldsymbol{z}_U + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{b}) \qquad \forall\, \boldsymbol{u} \in \mathbb{B}_k$$

$$\boldsymbol{L}^T\boldsymbol{\theta}_{\boldsymbol{u}} \leq \boldsymbol{g} + \boldsymbol{B}^T\boldsymbol{z}_U \qquad \forall\, \boldsymbol{u} \in \mathbb{B}_k$$

$$\boldsymbol{\theta}_{\boldsymbol{u}} \geq \boldsymbol{0} \qquad \forall\, \boldsymbol{u} \in \mathbb{B}_k$$

$$\boldsymbol{z}_U \in \mathcal{O}_S$$

- Since $\mathbb{B}_k \subset \mathbb{B}$, $\overline{\varphi}_k \geq \varphi^*$ and therefore an upper-bound for the *Full Problem*.
- Let $\boldsymbol{z}_{U,(k)}$ be the optimal solution for $\boldsymbol{z}_U$.

CCG Algorithm – Oracle

- Now, consider the following optimization problem:

$$\underline{\varphi}_k = \min_{\boldsymbol{z}_L \geq \boldsymbol{0},\ \boldsymbol{u} \in \mathbb{B}} \left\{ \boldsymbol{g}^T \boldsymbol{z}_L + \boldsymbol{z}_{U,(k)}^T \boldsymbol{B} \boldsymbol{z}_L \;\middle|\; \boldsymbol{L} \boldsymbol{z}_L \geq \boldsymbol{E} \boldsymbol{z}_{U,(k)} + \boldsymbol{F} \boldsymbol{u} + \boldsymbol{b} \right\}$$

- Since $\boldsymbol{z}_{U,(k)} \in \mathcal{O}_S$, then $\underline{\varphi}_k \leq \varphi^*$ and therefore a lower-bound for the *Full Problem*.
- Let $\boldsymbol{u}_k$ be the optimal solution for the binary vector $\boldsymbol{u}$.

CCG Algorithm – Description

- The pseudocode of the Column-and-Constraint Generation algorithm is:

Initialization: $UB \leftarrow +\infty, LB \leftarrow -\infty, k \leftarrow 1$ and $\varepsilon (> 0)$

while $UB - LB \geq \varepsilon$ **do**

Step 1: Solve *Master Problem* with $\mathbb{B}_k$. Store $\boldsymbol{z}_{U,(k)}$ and set $UB \leftarrow \overline{\varphi}_k$;

Step 2: Solve *Oracle Problem* with $\boldsymbol{z}_{U,(k)}$. Let $\boldsymbol{u}_k$ be the corresponding optimal binary vector. Set $LB \leftarrow \max\left\{LB, \underline{\varphi}_k\right\}$;

Step 3: Make $\mathbb{B}_{k+1} \leftarrow \mathbb{B}_k \cup \{\boldsymbol{u}_k\}$. Set $k \leftarrow k + 1$;

end

Return $\boldsymbol{z}_{U,(k-1)}$

Day-Ahead Bidding Problem – Small Example

- Assume the following small example.
- Strategic Player: fixed price bid; game only on quantity bids.

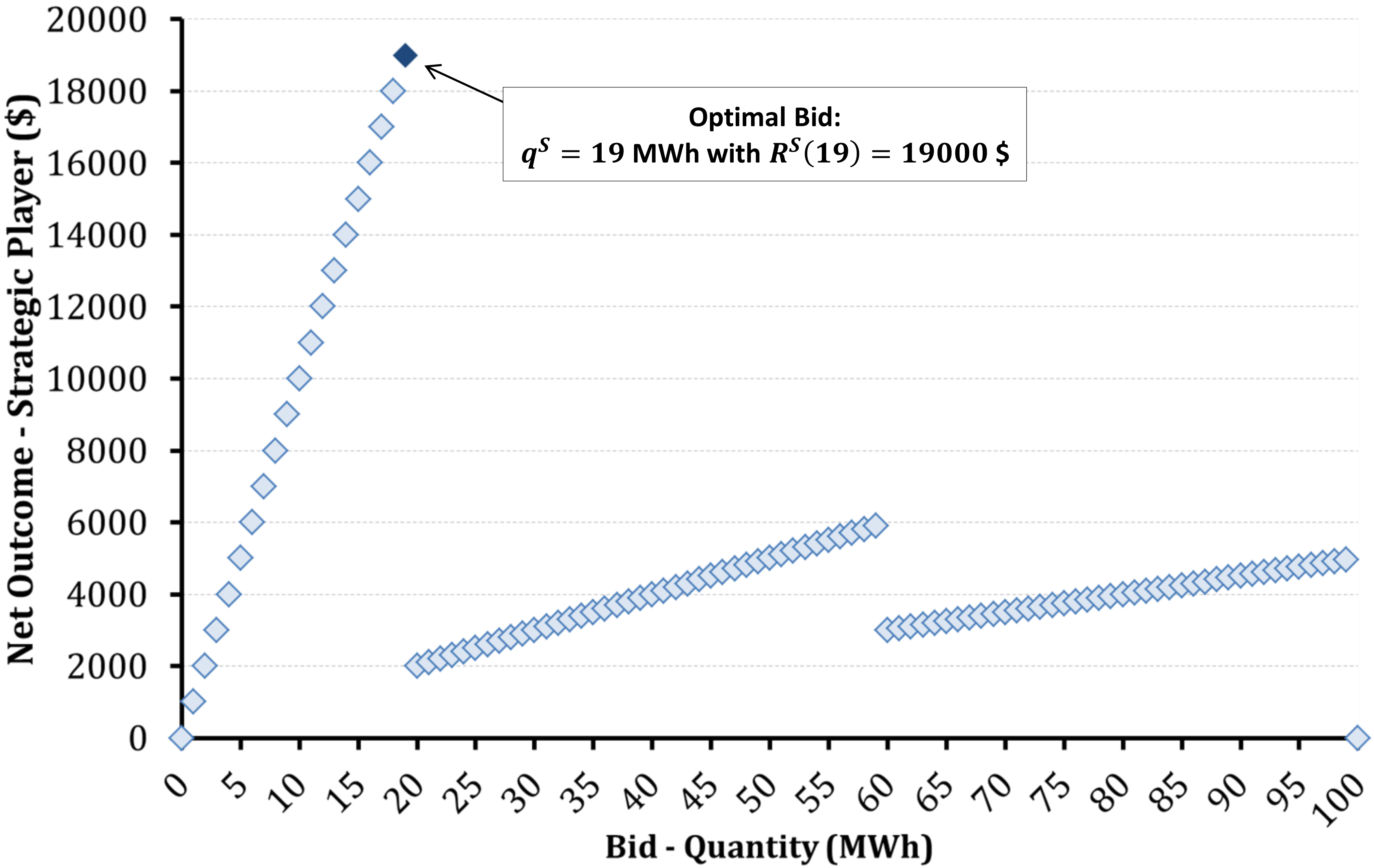
Strategic Unit	Price Bid ($/MWh)	Quantity Bid (MWh)	Cost ($/MWh)
# 1	0.00	[0:1:100]	0.00

- Rival Players: No uncertainty is assumed on rival bids (perfect information)

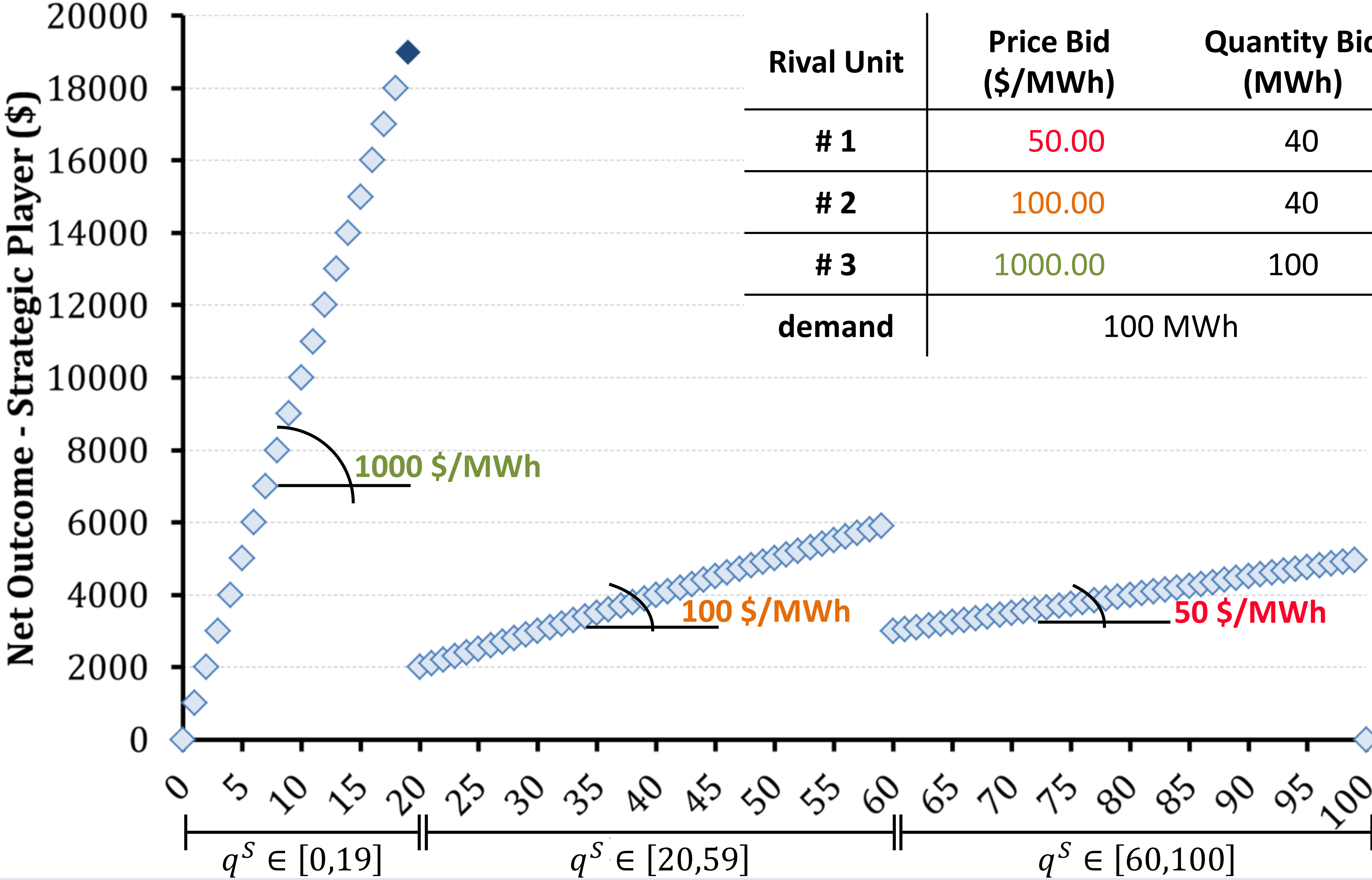
Rival Unit	Price Bid ($/MWh)	Quantity Bid (MWh)
# 1	50.00	40
# 2	100.00	40
# 3	1000.00	100

- Buyer: demand of $d = 100$ MWh

Small Example – Strategic Player Profit



Small Example – Strategic Player Profit



Rival Unit	Price Bid ($/MWh)	Quantity Bid (MWh)
# 1	50.00	40
# 2	100.00	40
# 3	1000.00	100
demand	100 MWh	

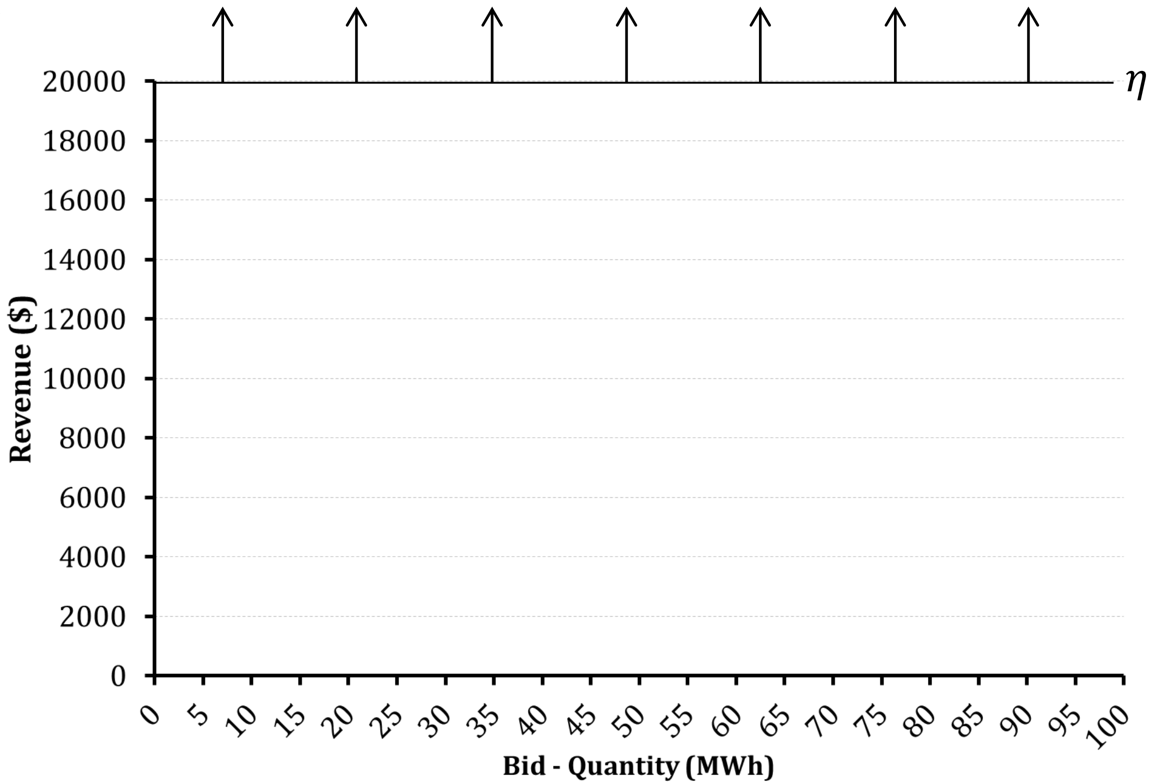
Small Example – CCGA Outline

- How is intuition behind the CCGA applied to this problem?
- Basically, each set of row and columm inserted in the *Master Problem* is related to a linear piece of the Strategic Player profit function.

$$\max_{\boldsymbol{p}_S, \boldsymbol{q}_S, \eta} \quad \eta$$

subject to:

$$(\boldsymbol{p}_S, \boldsymbol{q}_S) \in \mathcal{O}_S$$



Small Example – CCGA Outline

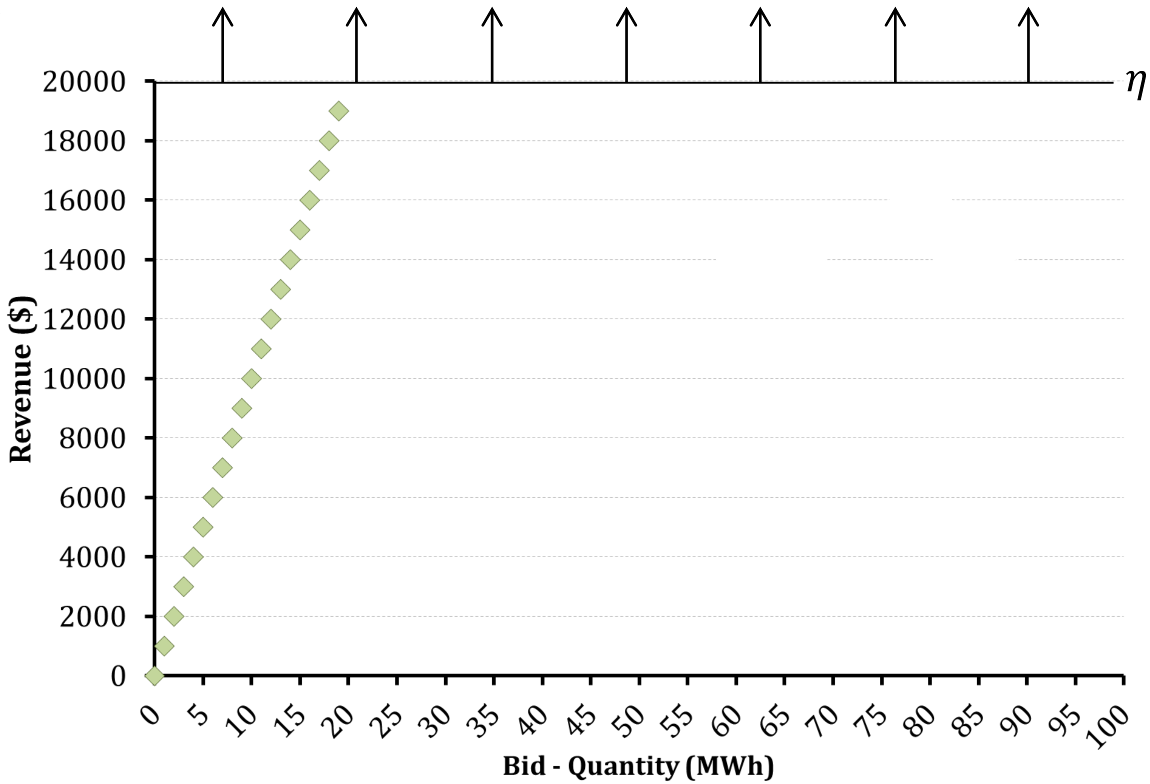
- How is intuition behind the CCGA applied to this problem?
- Basically, each set of row and columm inserted in the *Master Problem* is related to a linear piece of the Strategic Player profit function.

$$\max_{p_S, q_S, \eta} \; \eta$$

subject to:

$\eta \leq$ **Set of Row and Column – #1 Fixed $\mu^{(\#1)}$**

$$(p_S, q_S) \in \mathcal{O}_S$$



Small Example – CCGA Outline

- How is intuition behind the CCGA applied to this problem?
- Basically, each set of row and columm inserted in the *Master Problem* is related to a linear piece of the Strategic Player profit function.

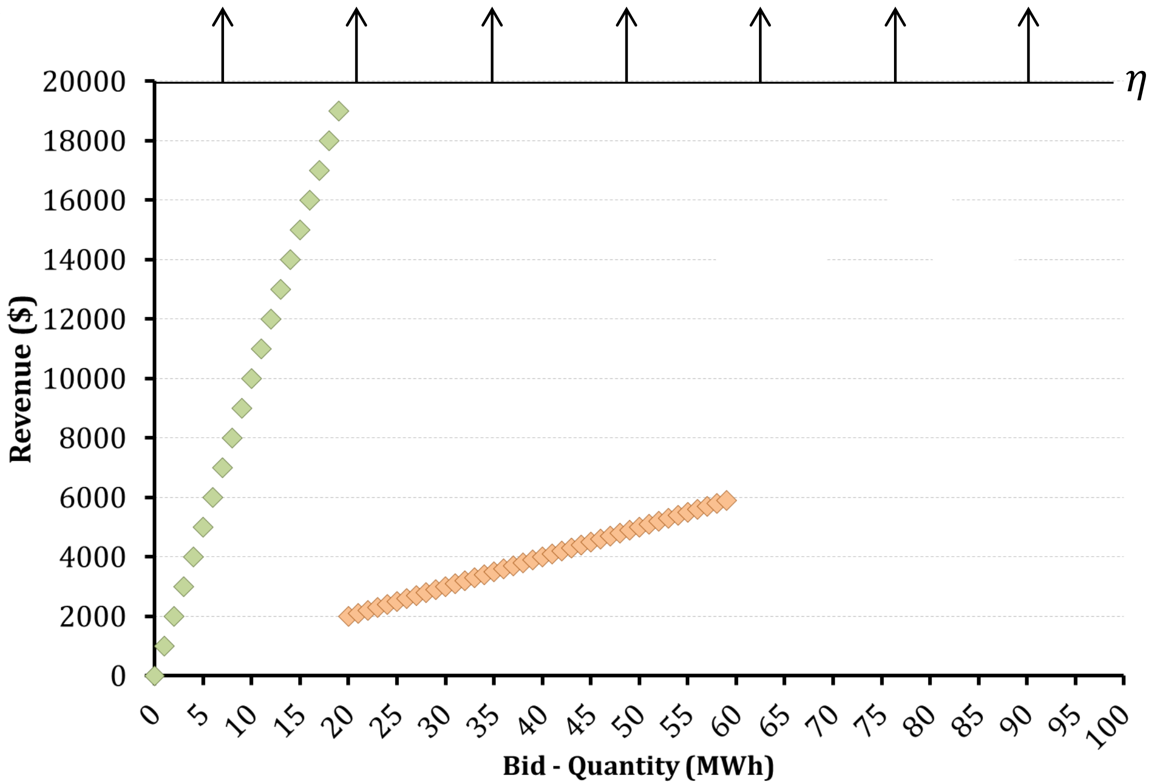
$$\max_{\boldsymbol{p}_S, \boldsymbol{q}_S, \eta} \ \eta$$

subject to:

$\eta \leq$ **Set of Row and Column – #1** **Fixed $\boldsymbol{\mu}^{(\#1)}$**

$\eta \leq$ **Set of Row and Column – #2** **Fixed $\boldsymbol{\mu}^{(\#2)}$**

$$(\boldsymbol{p}_S, \boldsymbol{q}_S) \in \mathcal{O}_S$$



Small Example – CCGA Outline

- How is intuition behind the CCGA applied to this problem?
- Basically, each set of row and columm inserted in the *Master Problem* is related to a linear piece of the Strategic Player profit function.

$$\max_{p_S, q_S, \eta} \ \eta$$

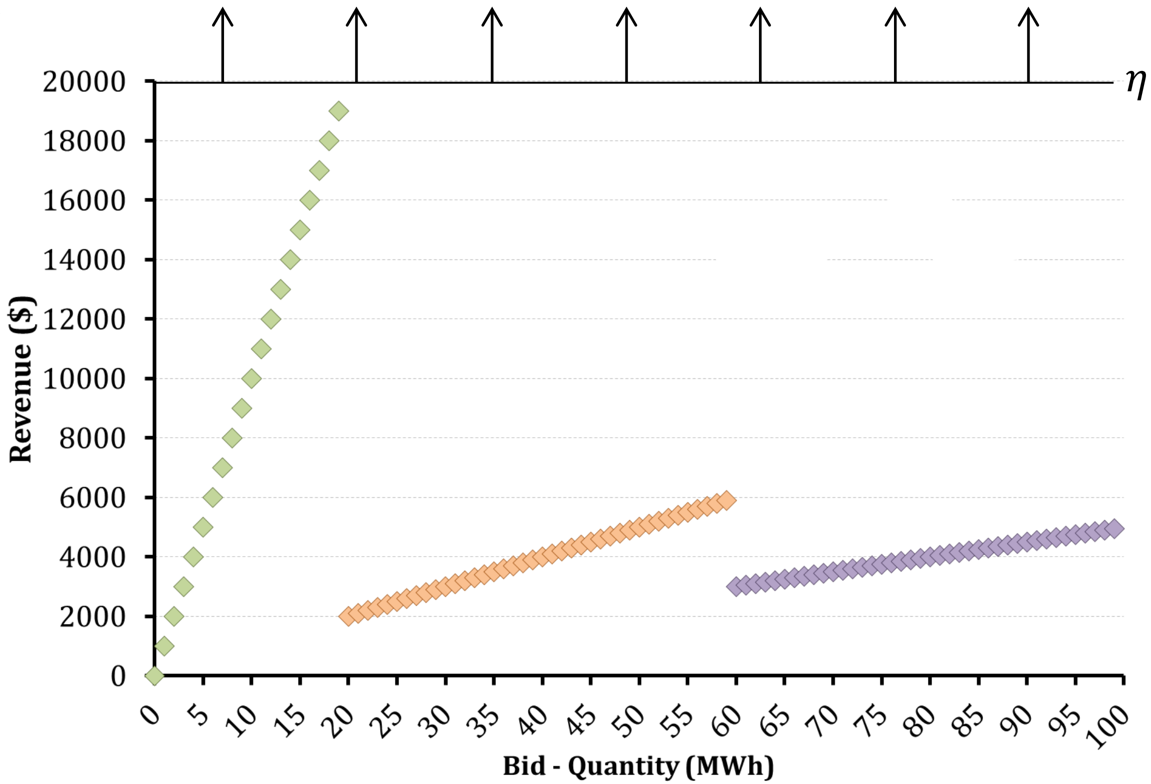
subject to:

$\eta \leq$ **Set of Row and Column – #1 Fixed $\boldsymbol{\mu}^{(\#1)}$**

$\eta \leq$ **Set of Row and Column – #2 Fixed $\boldsymbol{\mu}^{(\#2)}$**

$\eta \leq$ **Set of Row and Column – #3 Fixed $\boldsymbol{\mu}^{(\#3)}$**

$(\boldsymbol{p}_S, \boldsymbol{q}_S) \in \mathcal{O}_S$



Small Example – CCGA Outline

- How is intuition behind the CCGA applied to this problem?
- Basically, each set of row and columm inserted in the *Master Problem* is related to a linear piece of the Strategic Player profit function.

$$\max_{p_S, q_S, \eta} \ \eta$$

subject to:

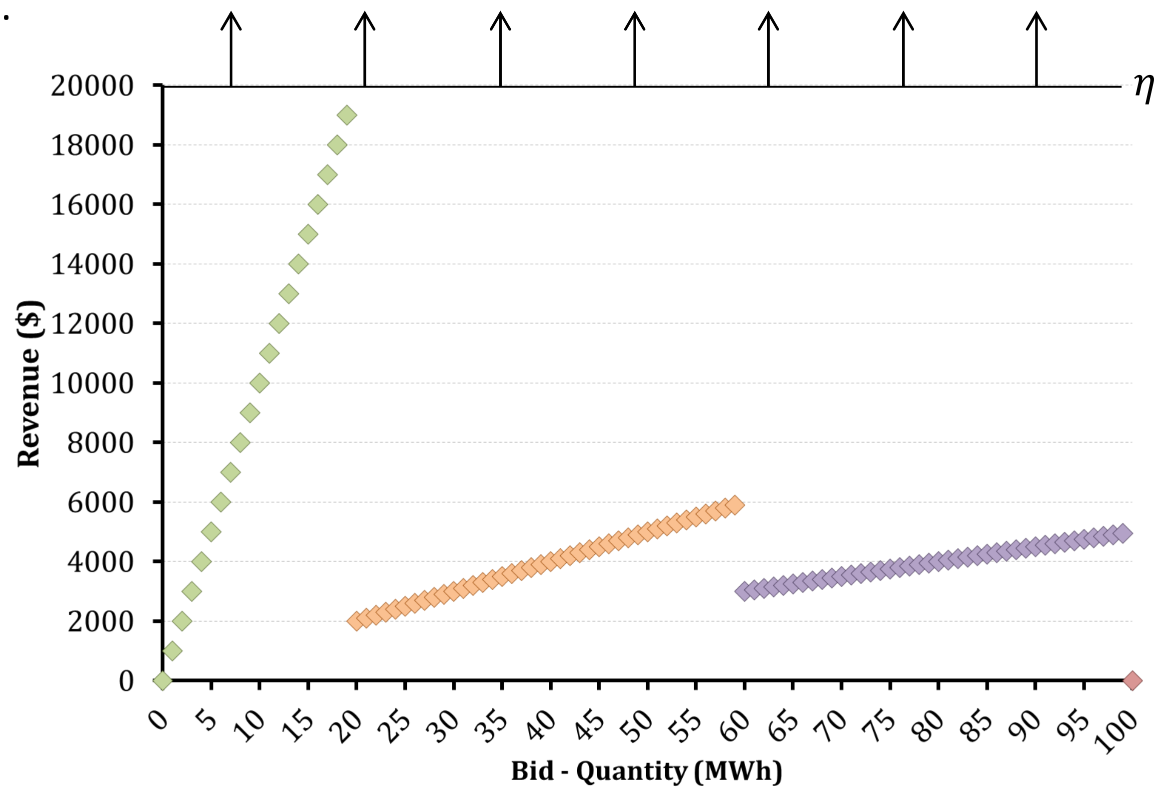
$\eta \leq$ **Set of Row and Column – #1** **Fixed $\mu^{(\#1)}$**

$\eta \leq$ **Set of Row and Column – #2** **Fixed $\mu^{(\#2)}$**

$\eta \leq$ **Set of Row and Column – #3** **Fixed $\mu^{(\#3)}$**

$\eta \leq$ **Set of Row and Column – #4** **Fixed $\mu^{(\#4)}$**

$(\boldsymbol{p}_S, \boldsymbol{q}_S) \in \mathcal{O}_S$



- At each new iteration, a new linear piece is "inserted" into the *Master Problem*.

Case Study – Uncertainty in Rival's Bid

- This second numerical experiment is composed by
  - The strategic player own two ($N_S = 2$) power units;
  - We consider 14 rival players ($N_R = 14$), devided in $N_R^{(PM)} = 4$ price makers and $N_R^{(PT)} = 10$ price takers;
  - A demand of $d = 195$.
- The strategic player feasible set is:

$$\mathcal{O}_S = \left\{ (\boldsymbol{p}_S, \boldsymbol{q}_S) \in \mathbb{Z}^{N_S} \times \mathbb{Z}^{N_S} \;\middle|\; \begin{array}{ll} 0 \le p_{S,j} \le \bar{p}_{S,j}, & \forall\, j \in \mathcal{N}_S \\ 0 \le q_{S,j} \le \bar{q}_{S,j}, & \forall\, j \in \mathcal{N}_S \end{array} \right\}$$

	Cost ($/MWh)	Capacity (MWh)	Price Cap ($/MWh)
Unit #1	10.00	50	60.00
Unit #2	30.00	20	60.00

Price Maker	$\underline{q}_R$ (MWh)	$\bar{q}_R$ (MWh)	c_R ($/MWh)
#1	10	40	60.00
#2	20	60	40.00
#3	5	40	45.00
#4	10	50	15.00

Price Taker	$\underline{q}_R$ (MWh)	$\bar{q}_R$ (MWh)	c_R ($/MWh)
#1	0	2	26.00
#2	0	3	48.00
#3	0	2	28.00
#4	0	3	35.00
#5	0	2	39.00
#6	0	2	32.00
#7	0	2	49.00
#8	0	3	54.00
#9	0	3	29.00
#10	0	3	28.00

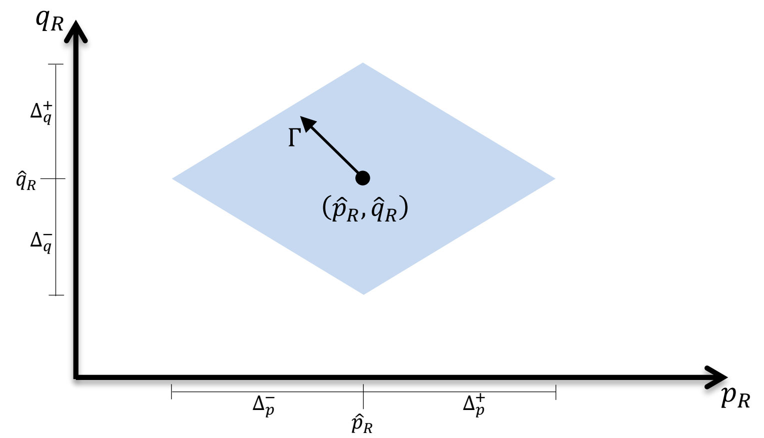
Case Study – Uncertainty in Rival's Bid

- In this numerical experiment, we assume available na estimative (Nash-Cournot equilibrium) of the rival's bid, denoted by $(\hat{\boldsymbol{p}}_R, \hat{\boldsymbol{q}}_R)$.
  - We argue that an accurate estimation of a joint probability distribution of market conditions that induce the equilibrium is a hard task.
  - Therefore, deviations from the nominal equilibrium point are very likely to be observed.

- Two sources of uncertainty were considered:
  1. Imprecision over the equilibrium point evaluation;
  2. Uncertainty related to the rival players' strategic action;

- The rival players' uncertainty set can be formulated as follows:

$$\mathcal{O}_R = \left\{ (\boldsymbol{p}_R, \boldsymbol{q}_R) \;\middle|\; \begin{array}{c} \exists\, \mathbf{0} \le \boldsymbol{v}^+, \boldsymbol{v}^-, \boldsymbol{w}^+, \boldsymbol{w}^- \le \mathbb{1} \;: \\ \boldsymbol{p}_R = \hat{\boldsymbol{p}}_R + \boldsymbol{\Delta}_{\boldsymbol{p}}^+ \boldsymbol{v}^+ - \boldsymbol{\Delta}_{\boldsymbol{p}}^- \boldsymbol{v}^- \\ \boldsymbol{q}_R = \hat{\boldsymbol{q}}_R + \boldsymbol{\Delta}_{\boldsymbol{q}}^+ \boldsymbol{w}^+ - \boldsymbol{\Delta}_{\boldsymbol{q}}^- \boldsymbol{w}^- \\ \mathbb{1}^T (\boldsymbol{v}^+ + \boldsymbol{v}^- + \boldsymbol{w}^+ + \boldsymbol{w}^-) \le \Gamma \end{array} \right\}$$



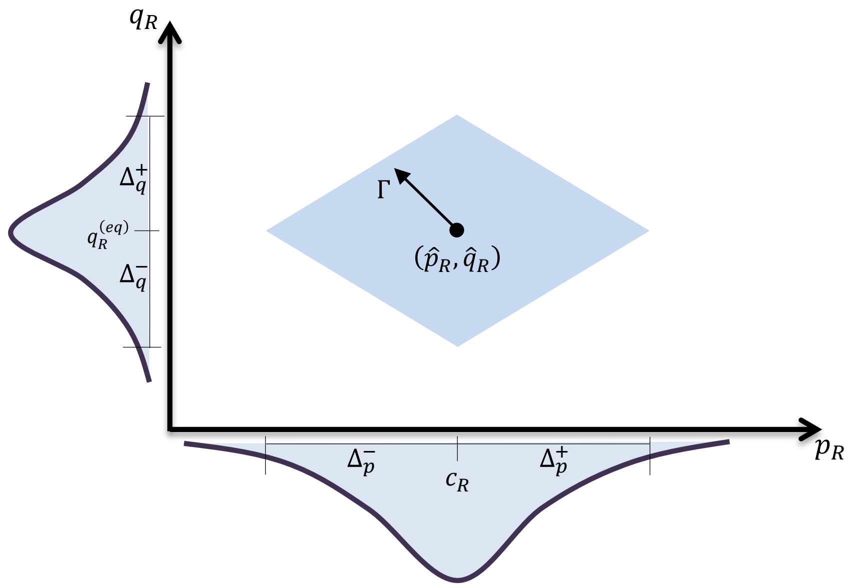
Case Study – Imprecision on Nominal Bids Estimation

- Let $\delta > 0$ quantify the level of imprecision on the Nash equilibrium evaluation.
- We can charaterize an imprecision on nominal bids estimation as follows:

i. $\left(\hat{p}_{R,i}, \Delta^{+}_{p_i}, \Delta^{-}_{p_i}\right) = \left(c_{R,i}, \delta c_{R,i}, \delta c_{R,i}\right), \quad \forall\, i \in \mathcal{N}_R;$

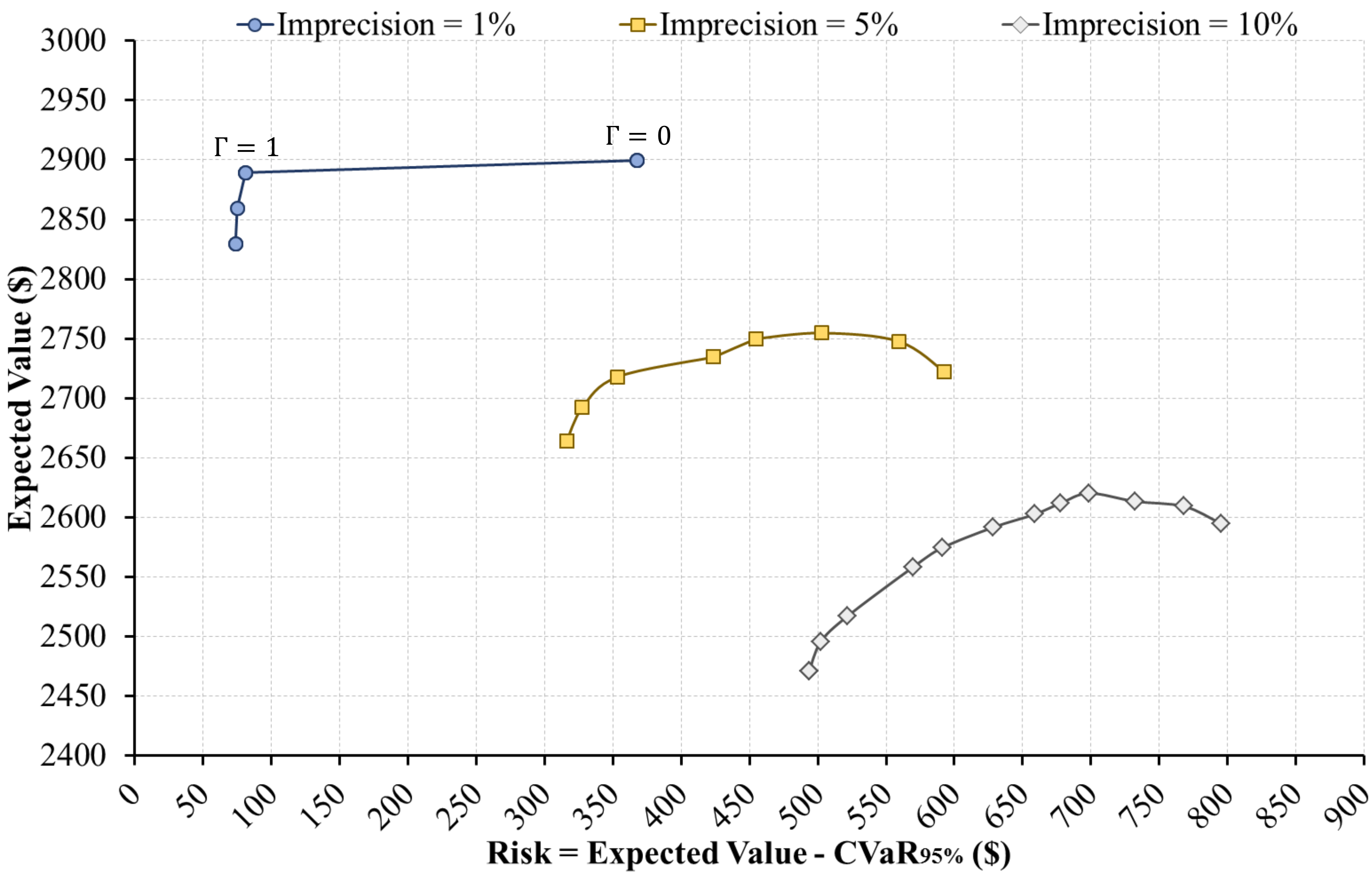
ii. $\left(\hat{q}_{R,i}, \Delta^{+}_{q_i}, \Delta^{-}_{q_i}\right) = \left(q^{(eq)}_{R,i}, \delta q^{(eq)}_{R,i}, \delta q^{(eq)}_{R,i}\right), \quad \forall\, i \in \mathcal{N}^{(PM)}_R;$

iii. $\left(\hat{p}_{R,i}, \Delta^{+}_{p_i}, \Delta^{-}_{p_i}\right) = \left(q^{(eq)}_{R,i}, 0{,}0\right), \quad \forall\, i \in \mathcal{N}^{(PT)}_R;$



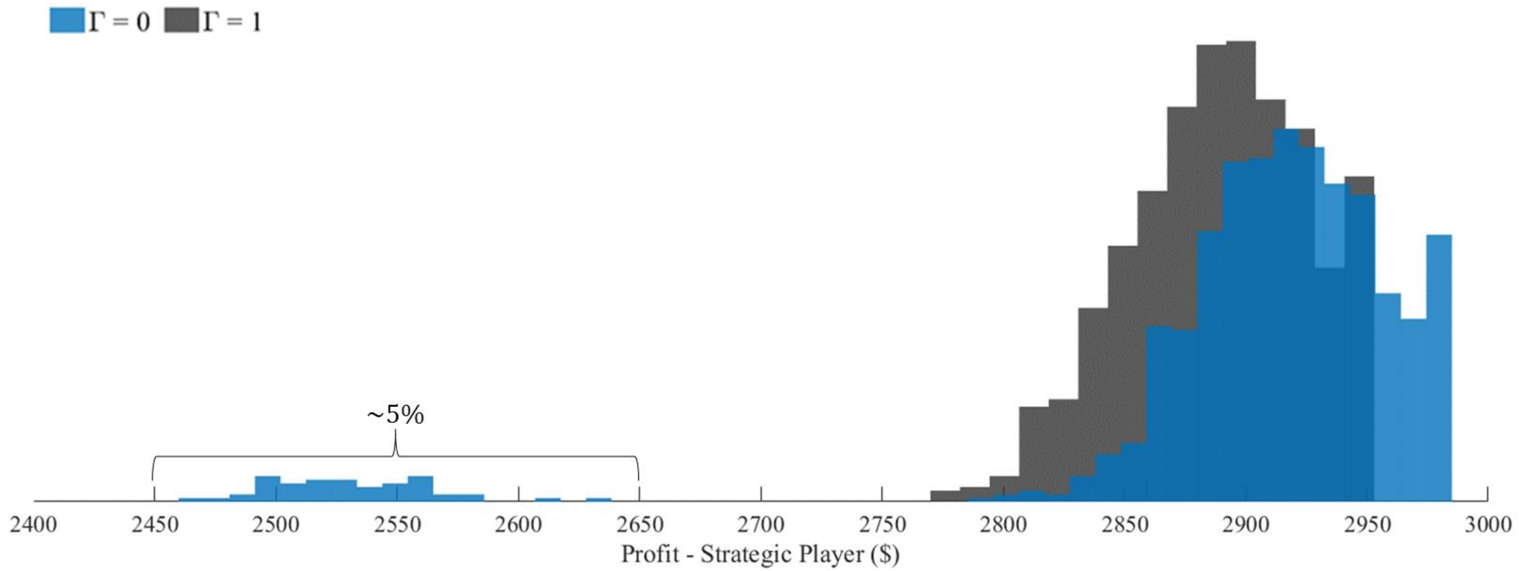
Case Study – Imprecision on Nominal Bids Estimation

- Let $\delta > 0$ quantify the level of imprecision on the Nash equilibrium evaluation.



Case Study – Imprecision on Nominal Bids Estimation

- Let $\delta > 0$ quantify the level of imprecision on the Nash equilibrium evaluation.
  - Net revenue distribution assuming $\delta = 1\%$ for conservativeness levels of $\Gamma \in \{0, 1\}$.



- **We highlight that even in market conditions in which the equilibrium can be estimated within an 1% imprecision, bidding the equilibrium can induce a significant risk for the strategic player.**
- **The proposed robust model identify a bidding strategy with a slight reduction in expected revenue, but with significantly less risk.**

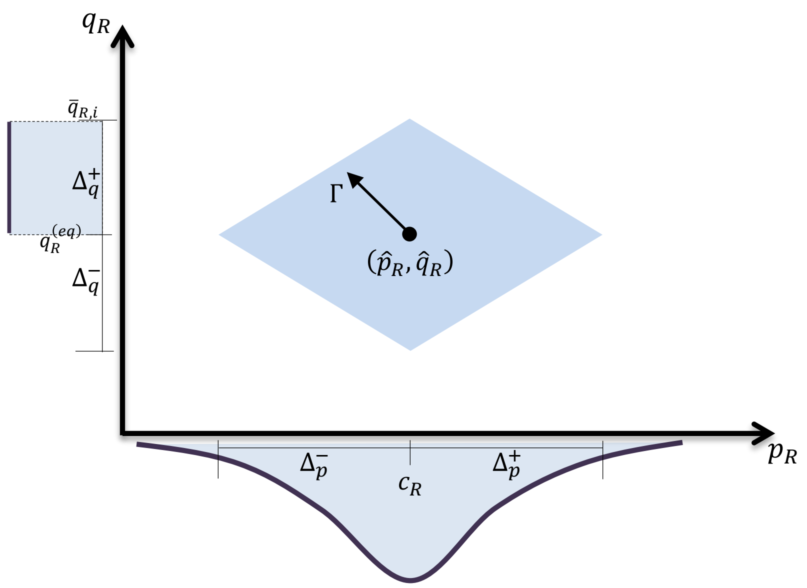
Case Study – Uncertainty Rivals' (Quantity) Bid

- Let $\zeta > 0$ quantify the level of uncertainty on rivals' bid and fix $\delta = 30\%$.
- For this case, we can construct the uncertainty set as follows:

i. $\left(\hat{p}_{R,i}, \Delta^{+}_{p_i}, \Delta^{-}_{p_i}\right) = \left(c_{R,i}, \delta c_{R,i}, \delta c_{R,i}\right), \quad \forall\, i \in \mathcal{N}_R;$

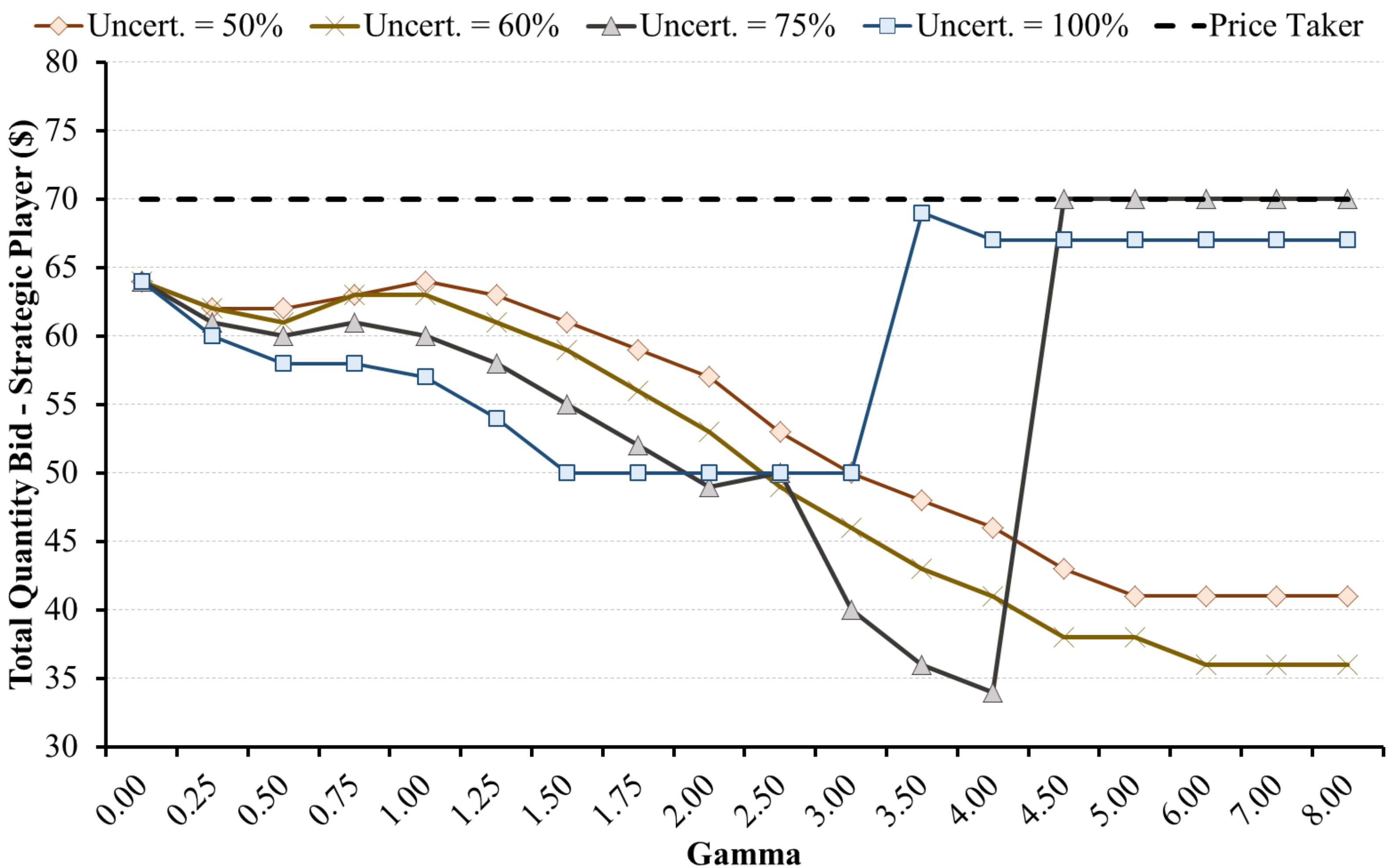
ii. $\left(\hat{q}_{R,i}, \Delta^{+}_{q_i}, \Delta^{-}_{q_i}\right) = \left(q^{(eq)}_{R,i}, \zeta\left(\bar{q}_{R,i} - q^{(eq)}_{R,i}\right), 0\right), \quad \forall\, i \in \mathcal{N}^{(PM)}_R;$

iii. $\left(\hat{p}_{R,i}, \Delta^{+}_{p_i}, \Delta^{-}_{p_i}\right) = \left(q^{(eq)}_{R,i}, 0, 0\right), \quad \forall\, i \in \mathcal{N}^{(PT)}_R;$



Case Study – Uncertainty Rivals' (Quantity) Bid

- Let $\zeta > 0$ quantify the level of uncertainty on rivals' bid and fix $\delta = 30\%$.



Thank You

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Bruno Fanzeres, Shabbir Ahmed, and Alexandre Street, "Robust Strategic Bidding in Auction-Based Markets," *European Journal of Operational Research*, vol. 272, no. 3, pp. 1158-1172, Feb. 2019.